A Dynamic Analysis of Growth via Acquisition

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Abstract. Firms can grow through internal investment or through acquisition. While internal growth takes time, an acquisition provides cash flows immediately. The opportunity to grow internally affects the price of an acquisition as it is a fall-back option for the acquirer should negotiations break down. Assuming investors do not have full information about the time a firm requires to grow internally, acquirers earn positive returns before the announcement of an acquisition, and there are negative stock price reactions to acquisition announcements. This research provides predictions about how pre-announcement price run-up and negative announcement returns relate to integration costs and synergies from acquisition.

JEL Classification: G31, G34

1. Introduction

When a firm wants to expand, it needs additional assets. In the case of a geographic expansion, this typically includes purchasing land, buildings, and machines, training new employees, and building relationships with new customers. Such an expansion requires both a substantial financial commitment and time for the investment to generate cash flows.

A firm can also acquire additional assets by buying an established business, whether an entire firm or a division of a firm. This way the firm can access a new market more quickly, as it will benefit from the investments of the previous owners. For a pharmaceutical company to expand into a new area of drug development, for example, it would take years of research before a new product could be brought to market. Rather than make these internal investments itself, the firm can acquire another firm already operating in that area and take advantage of that firm’s accumulated knowledge.¹

We analyze this fundamental trade-off between internal growth and growth via acquisition. Internal growth and growth via acquisition are modelled as

¹ As an example, P. Hug of Roche is quoted saying, “If you can’t refuel with new innovative drugs [internally], you go outside” [Financial Times, May 27, 2005].

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opportunities to acquire a cash flow generating set of assets. If the firm decides to grow internally, it must make two investments, with some time between the two before realizing any benefits. If the firm makes an acquisition, we assume instantaneous access to cash flows after paying integration costs.

Although these two alternatives for growth are normally mutually exclusive, there is an important connection between them. Before the firm makes an acquisition, we assume that the firm can always choose to grow internally, so internal investment is a fall-back strategy. Internal growth thus influences both the decision to acquire and the acquisition price, given the value of the alternative to invest internally, which we show is inversely related to the price paid in the acquisition.2 Whenever an acquisition occurs at a value higher than the optimal threshold value to invest internally, the acquirer can try to reduce the price paid by making the acquisition sooner. Earlier acquisition increases the value of the internal growth option, and thus the buyer’s negotiating power. This strategic action can push the acquisition threshold below the level that maximizes the social surplus obtained in the transaction. This happens when the costs of integrating the acquired unit are high, or when there are relatively few synergies from the acquisition. In this case, although there is no uncertainty about these parameters, if outside investors are imperfectly informed about the profitability of the internal investment, but might infer it from the moment of the acquisition, then an acquisition generally sends a negative signal that affects the stock price of the acquiring firm.

An interest in preserving the value of the internal investment and using it wisely in negotiations motivates the acquirer to approach the seller. For a wide range of parameter values, an acquirer-initiated transaction results in a negative stock price reaction. For the remaining parameter values, the negotiations are initiated either by the buyer or by the seller. In this case, the stock price reaction is zero on average. That is, there are significantly different announcement returns whether the acquisition is initiated by the buyer or the seller.

The model also shows that the value of the acquirer increases for some time before an acquisition is announced. This happens because the acquirer has the option to either implement the internal growth strategy or the acquisition, and as time passes without announcement of an acquisition, outside investors increase their estimate of the value of the internal growth opportunity. The extent of the price run-up is negatively related to the acquirer’s announcement return.

We also find that acquisitions with low levels of synergies or relatively high integration costs do not maximize the social surplus. This is because, in these cases, the

2 The importance of profitable alternatives for acquisition strategy is noted by H. de Castries, CEO of Axa: “If we do not want to become a prisoner to acquisitions, we need to have strong organic growth” [Financial Times, September 23, 2005].
value of the option to grow internally deteriorates with time, and to prevent it from happening, the acquirer makes the acquisition too early from an efficiency standpoint. The result that an acquisition is not always initiated at a level that maximizes the overall surplus to society is in contrast to Jovanovic and Braguinsky (2004), who find that in a competitive industry mergers are not just privately, but also socially efficient. The social inefficiency increases with the level of bargaining power of the seller.

Wealth effects associated with the dynamics of the stock price in takeover contests have been the subject of some discussion. Harford (1999) and Ang and Cheng (2003) find that the stock of acquiring firms performs well in the years before an acquisition. Schwert (2000) and Andrade et al. (2001) find a negative abnormal price reaction to the announcement of a bid. Why then would firms decide to proceed with an acquisition when in general the market reacts negatively to such actions? Explanations have so far been confined to agency conflicts, errors of judgment, or market irrationality. For example, Roll (1986) argues that managers of bidder firms incorrectly assess the value of the combined firms. Shleifer and Vishny (1989) claim managers overinvest in assets that suit their skills in order to entrench themselves. Shleifer and Vishny (2003) argue later that mergers occur when managers take advantage of the opportunities created when inefficient financial markets value some firms incorrectly. Our model shows there may be price run-ups before and price declines upon the announcement of an acquisition in rational markets in the absence of any agency conflicts.

McCardle and Viswanathan (1994) and Jovanovic and Braguinsky (2004) also analyze the trade-off between internal investment and acquisitions. McCardle and Viswanathan (1994) model a duopoly with one potential entrant. The entrant can achieve market entry either through its separate entity, which increases the number of competitors, or through an acquisition. The decision to enter via acquisition signals a high cost of entry, and may cause negative announcement returns for the acquirer. In horizontal mergers, Jovanovic and Braguinsky (2004) show negative announcement effects even when mergers are individually and socially efficient.

Despite these contributions to our understanding of acquisition announcement effects, it is difficult to use a static approach to explain the dynamics of stock returns around the announcement of acquisitions, for at least three reasons: (1) A static model does not allow for recognition of important differences between internal and external investment, such as that internal growth takes more time. We show that this feature has a profound effect on the timing of acquisitions and on whether announcement returns are positive or negative.

(2) A static model forces any investment of the firm to occur at one particular time. This is typically not the optimal choice, which has important implications for learning by outside investors. Even when investors anticipate an acquisition rather
than internal growth, announcement returns may be negative if the timing of the acquisition surprises investors. Identifying the conditions under which this occurs allows us to derive a number of novel empirical predictions about the effect of acquisition characteristics on announcement returns.

(3) A static model does not allow us to draw conclusions about stock market returns before or after acquisitions. Consistent with the empirical evidence, our model is able to generate positive stock market returns for the acquirer before an acquisition is announced.

There is a growing literature that studies acquisition strategies in a dynamic context. Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004) study a model of acquisitions that assumes financial markets may misvalue both acquirer and target. They show that this can lead to a correlation between merger activity and market valuation. Lambrecht (2004) shows that this can also occur when mergers increase market power or are designed to generate economies of scale. Gorton et al. (2005) note that mergers are a defensive instrument for managers trying to avoid being taken over. Lambrecht and Myers (2007) show that takeovers are a more efficient mechanism for industry contraction than firm or plant closures when managers’ interests conflict with shareholders interests. Hackbarth and Morellec (2008) develop a real options model to analyze the behavior of stock returns and present evidence on the dynamics of firm-level betas before, around, and after mergers.

Morellec and Zhdanov (2005) examine the effect of multiple bidders and imperfect information on takeover activity. When investors are uncertain about the synergies potential acquisitions create, competition for targets may lead to negative price reactions upon acquisition announcements. Our model complements this result. We show that competition is not necessary to generate negative announcement returns and derive different empirical predictions. Also, including the opportunity of internal investment demonstrates, in contrast to Lambrecht (2004) and Morellec and Zhdanov (2005), that acquisitions frequently take place earlier than socially optimal.

The remainder of the paper is structured as follows Section 2 contains the model in which we derive the acquisition price endogenously as the outcome of a bargaining game between acquirer and seller when the acquirer initiates the negotiations. Section 3 analyzes the effect of the characteristics of the opportunity to grow internally on the acquisition strategy. Section 4 shows that the model generates a price run-up of the acquirer’s stock prior to an acquisition and its subsequent decline upon the announcement of the acquisition. Section 5 looks at the case when either party can initiate the transaction. It shows that buyer-initiated transactions lead to more significant announcement returns on average than seller-initiated transactions. Section 6 discusses the model’s empirical implications. Section 7 concludes.
2. A Dynamic Model of Acquisitions

Suppose a risk-neutral firm is planning to obtain a set of assets of value $V$. The value of the set of assets follows a geometric Brownian motion:

$$\frac{dV}{V} = (\mu - \delta) dt + \sigma dZ,$$

where $\mu - \delta < r$ is the expected percentage change of $V$ (\mu is the total expected rate of returns, and $\delta$ is the payout rate to securityholders); \sigma is the volatility rate, which is assumed to be constant; and $dZ$ is the increment of a standard Brownian motion. A risk-free rate is fixed at the rate $r$. At the beginning, the firm must choose between two mutually exclusive alternatives: 1) it can either obtain the set of assets by acquiring another business, or 2) assemble the set by producing or purchasing the assets individually.

We assume throughout that the firm is all-equity financed, and that its managers act in the interest of equityholders who pay for the investment cost in both cases. Under these assumptions, there are no agency conflicts.

2.1 ACQUISITION

By acquiring another business, a firm buys a set of assets already in place and already generating cash flows. Buying assets as a package allows the firm to produce cash flows sooner than it could if it had to buy the assets separately. To capture the notion of quick cash flow generation, we assume acquisition will give the firm immediate access to the set of assets. Acquiring the assets requires an investment of $kA$, which has two components: (1) a fixed deadweight cost, $F > 0$, which represents the expenses of integrating the new business entity and is given exogenously in the model, and (2) the acquisition price, $pA$, which is determined endogenously by bargaining between the acquirer and the shareholders of the seller. The acquisition price is not constant but depends on the value of the asset at the time of the acquisition. When acquisition timing is flexible, real options theory asserts that it is not optimal to invest when the asset value is equal to the investment amount, but rather when $V$ is equal to some critical value that is higher than the investment amount. We denote the critical value of the set of assets at which the
acquisition takes place as $V^A_G$. $V^A_G*$ indicates the optimal value of $V^A_G$ at which the value of the opportunity to acquire is maximized. The value of the acquisition opportunity is denoted as $v^A(V)$ and the optimal $v^A(V)$ as $v^A(V)$.

The value of the opportunity to acquire at any value of $V^A_G > V$ is:

$$v^A(V) = \left[ V^A_G - k^A (V^A_G) \right] \left( \frac{V}{V^A_G} \right)^\gamma,$$

(2)

where

$$k^A (V^A_G) = F + p^A (V^A_G),$$

(3)

and $\gamma$ is a positive root of the quadratic equation

$$\frac{1}{2} \sigma^2 \gamma (\gamma - 1) + (r - \delta) - r$$

and is given by:

$$\gamma = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} + \sqrt{\left[ \frac{(r - \delta)}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2}}.$$

(4)

The term $V^A_G - k^A (V^A_G)$ in (2) is the net benefit when the firm acquires the set of assets at the value $V^A_G$, while $(V/V^A_G)^\gamma$ can be interpreted as the risk-neutral probability that the current asset value $V$ will reach the level $V^A_G$. Naturally, the value of the acquisition declines with $k^A (V^A_G)$ as a higher $k^A (V^A_G)$ implies lower profits from the acquisition. The derivation of (2)–(4) is standard in the real options literature [see McDonald and Siegel (1984) and Dixit and Pindyck, Chapter 6 (1994)].

2.2 ENDOGENOUS ACQUISITION PRICE

The acquisition price $p^A (V^A_G)$ is determined by bargaining between the acquirer and the shareholders of the seller. We assume first that the acquiring firm decides if and when to enter into negotiations with the seller’s shareholders. It is typically beneficial for a bidder to hide its plans as long as possible to reduce the risk of a preemptive competitive bid. The value $V$ is not contractible, for example, due to the lack of verifiability in court. Thus, it is impossible to negotiate a transaction ex ante that is to be executed if $V$ reaches a certain value in the future at specified terms.

For simplicity, we assume that managers of the seller represent the interests of its shareholders, and all shareholders follow an agreement reached between the acquiring firm and the seller’s managers.

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6 The seller may also initiate the negotiations. We analyze this case in Section 5.

7 In the case of a takeover, one could also assume that the firm negotiates with the shareholder whose vote allows the acquirer to control the firm and freeze out the non-tendering shareholders at the negotiated price. For a description of freezeout laws and practices, see Amihud et al. (2004).
Once the acquirer and the seller begin negotiations, their outcome is determined
by both parties’ outside options as well as the distribution of bargaining power. We
treat the distribution of bargaining power as exogenous here. The seller’s bargaining
power is denoted by $\rho \in [0, 1]$. We assume that acquisition negotiations are possible
only once. If the parties do not agree on the terms of the acquisition, resuming
acquisition talks later is not possible.

The assumption of one-time negotiations is reasonable as frequently a consider-
able part of the synergies between acquirer and acquiree arise due to the element
of surprise associated with the acquisition. For example, when negotiations or their
failure become public, information is revealed to competitors who may be able
to take actions rendering a future acquisition unprofitable (or less profitable than
an immediate agreement). The party benefiting from an early agreement has the
incentive to make such information publicly available as it causes a procrastinating
strategy of the other party to become suboptimal. Another way of eliminating future
negotiations is to commit to taking the alternative course of action should negotia-
tions fail. For example, the acquirer can either engage in contractual commitments
that cause an internal strategy to be optimal upon the failure of negotiations. Possibly
more prevalent is the use of reputational capital vis a vis employees, customers,
suppliers or investors to ascertain that internal growth is bound to take place should
negotiations fail. For example in case of a geographic expansion, the acquirer may
publicly announce its entering of the new regional market, de facto, committing it
to such a move (either via acquisition or internal investment) is it to avoid the cost
of losing reputational capital with various groups of stakeholders. Again, the party
benefiting from an immediate agreement relative to a later one has the incentive to
pursue such activities.

As we will demonstrate below, the assumption that negotiations take place once
combined with the impossibility of striking a deal ex ante implies that a surplus-
maximising outcome cannot be guaranteed.

To complete an acquisition, the acquirer and the seller must agree on the price of
the acquisition, $p^A(V^A_G)$. Otherwise the deal falls through, and each party is left with
its outside options. If an acquisition creates surplus at the time that negotiations take
place, the parties reach an agreement, and $p^A(V^A_G)$ is assumed to be determined
by the Nash bargaining solution. When the outside options of the acquirer are
denoted by $d^A(V^A_G)$ and the outside options of the seller as $d^S(V^A_G)$, the equilibrium
acquisition price is given by

$$
p^A(V^A_G) = d^S(V^A_G) + \rho \left[ V^A_G - F - d^A(V^A_G) - d^S(V^A_G) \right].
$$

(5)

2.3 OUTSIDE OPTION OF THE SELLER

To analyze the optimal acquisition threshold, the values of the parties’ outside
options need to be characterized. When the set of assets is not sold to the acquirer,
it is employed in the next best alternative, and its value is assumed to be \( aV \), where \( a \in (0, 1) \). This assumes that the assets of the seller when combined with those of the buyer increase in value by \((1 - a) V\) over the next-best alternative. At the time of the negotiations, the seller’s outside option is \( d^S = aV_G^A \). The seller’s expected wealth for \( V < V_G^A \), \( s(V) \), is given by

\[
s(V) = aV + \left( p^A \left( V_G^A \right) - aV_G^A \right) \left( \frac{V}{V_G^A} \right)^\gamma. \tag{6}
\]

where \( aV \) represents the value of the set of assets when it is used in the next-best alternative, while the second term can be interpreted as an option to acquire the assets \( aV_G^A \) for the payment \( p^A \left( V_G^A \right) \) if the value reaches \( V_G^A \). The term \( \left( \frac{V}{V_G^A} \right)^\gamma \) can be interpreted as the (discounted) risk-neutral probability that \( V_G^A \) is reached. It is clear from this equation that the seller’s strategy is to sell the assets to the acquirer only when \( p^A \left( V_G^A \right) \geq aV_G^A \). See the appendix for the derivation of Equation (6).

### 2.4 Internal Growth as the Outside Option of the Acquirer

If the acquiring firm and the seller do not reach an agreement, the acquirer has the opportunity to assemble the assets required to grow through individual investments. This opportunity to grow internally is itself an option, assuming the acquirer has the flexibility to decide if and when to make the investment. We assume that acquiring and growing internally are mutually exclusive strategies. This means that even if the inferior alternative has a lower optimal starting threshold, it will not be implemented. More specifically, the firm does not start to grow internally and later acquires, and vice versa. Then, the value of the option to grow internally at the time of negotiations represents the acquirer’s outside option in the bargaining game. For comparability, we assume the value of the set of assets when growing internally is identical to the value in the case of an acquisition. We denote the value of the option to grow internally by \( v^O(V) \). When the firm decides to invest internally, it needs to assemble the set of assets by itself. This takes time. Thus, it cannot obtain immediate access to the cash flow of the complete set of assets.

We assume the firm’s investment takes place in two stages. In the first stage, the firm can obtain a portion \( \theta \in (0, 1) \) of the set of assets. The first-stage investment is assumed to be proportional to the total investment, i.e., the first-stage investment is \( \theta \kappa^O \), where \( \kappa^O \) is the total investment in internal growth. We denote any arbitrarily chosen level of the asset value that the firm executes in the first-stage investment as \( V^O_G \). When \( V^O_G \) is chosen optimally to maximize the value of internal growth, we denote it by \( V^O_G^* \). The optimal value of \( v^O(V) \) is denoted by \( v^O^*(V) \).

A first-stage investment allows the firm to proceed to the second investment stage. In the second stage, the firm obtains the remainder of the asset, \( 1 - \theta \), for an
investment of \((1 - \theta)k^O\). To represent that internal growth is slower than growth via acquisition, we assume the asset value has to increase to \(\beta V^O^*\), with \(\beta > 1\), before the second-stage investment can be made. That is, some time has to elapse between the first and the second investment stage. Because \(V\) is stochastic, the shortest time between the two investments, \(T\), is a random variable – a stopping time for the geometric Brownian motion. The expected length of the delay, \(T\), provided that \((\mu - \delta) \geq \frac{\sigma^2}{2}\), is given by

\[
E[T|V^O] = \frac{1}{(\mu - \delta) - \frac{\sigma^2}{2}} \ln(\beta).
\]

Note that the distribution of \(T\) is independent of the first-stage investment threshold, \(V^O\), which implies that \(E[T|V^O]\) is also unaffected by \(V^O\). An increase in \(\beta\) implies a longer expected time delay. Also, notice that in calculating \(E[T|V^O]\), we use a real instead of a risk-neutral measure in order to obtain the expected time delay in actual calendar time, since it is more instructive and easier to interpret. For example for \((\mu - \delta) = 0.2, \sigma = 0.10, and \beta = 1.5\), 2.07 years is anticipated to complete the internal growth. To calculate the expected time delay using a risk-neutral measure, replace \((\mu - \delta)\) with \((r - \delta)\). Naturally, the latter is longer than the former.

The maximum value of the option to grow internally, \(v^O^*(V)\), is given in Lemma 1:

**Lemma 1.** The maximum value of the option to grow internally is

\[
v^O^*(V) = \begin{cases} 
\theta(V - k^O) + (1 - \theta)(\beta V - k^O)(\frac{1}{\beta})^\gamma & \text{for } V > V^O^* \\
\theta(V^O^* - k^O)(\frac{V}{V^O})^\gamma + (1 - \theta)(\beta V^O^* - k^O)(\frac{V}{\beta V^O})^\gamma & \text{for } V \leq V^O^*, 
\end{cases}
\]

where \(\gamma\) is given in (4). The optimal first-stage investment threshold is given by

\[
V^O^* = \frac{\gamma \theta + \beta^{-\gamma}(1 - \theta)}{\gamma - 1 \theta + \beta^{1-\gamma}(1 - \theta)} k^O.
\]

The optimal second-stage investment threshold is \(\beta V^O^*\) for \(V \leq V^O^*\) and \(\beta V\) for \(V > V^O^*\).

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8 Alternative ways to model the time between initial investment and access to cash flows include assuming a limited investment rate (see Majd and Pindyck, 1987, Milne and Whalley, 2000, 2001) or an exogenously specified time lag between an initial and a final investment (see Bar-Ilan and Strange, 1996). One difficulty with such time-to-build or gestation period models is that no closed-form solutions are available. Numerical analysis indicates, however, that the main results are not affected if one of these alternative models is chosen.
When \( V \) is lower than \( V^*_G \), the value of internal growth opportunity is the sum of the value of the real option of the first-stage investment, \( \theta(V^*_G - k)^{\gamma} \), and the second-stage investment, \((1 - \theta)(\beta V^*_G - k)^{\gamma} \beta^{V^*_G - k} \gamma \), where \( \theta(V^*_G - k) \) and \((1 - \theta)(\beta V^*_G - k) \beta^{V^*_G - k} \gamma \) are the net benefits of the investment in each stage, and \( \gamma^{V^*_G - k} \beta^{V^*_G - k} \gamma \) are the corresponding risk-neutral probabilities that \( V \) will reach the investment threshold in each stage.

When \( V \) is greater than \( V^*_G \), the option to grow internally is exercised immediately, so \( V^*_G = V \), and the second-stage investment threshold becomes \( \beta V \), and the term \( \gamma^{V^*_G - k} \beta^{V^*_G - k} \gamma \) becomes 1, and \( \gamma^{V^*_G - k} \beta^{V^*_G - k} \gamma \) becomes \((1/\beta)^y \), hence \( V^*_G(V) = \theta(V - k) + (1 - \theta)(\beta V - k) \beta^{V^*_G - k} \gamma \).

At the time acquisition negotiations take place, the value of growing internally is \( d^A(V^*_G) = v^{O*}(V^*_G) \). Notice that in a special case in which \( \beta = 1 \), or there is no time delay, \( V^*_G = \gamma^{V^*_G - k} \), and \( v^{O*}(V) = (V^*_G - k)^{\gamma} \beta^{V^*_G - k} \gamma \) for \( V \leq V^*_G \), a standard result for a real option threshold and valuation, as in Dixit and Pindyck (1994).

The second-stage delay parameter \( \beta \) plays an important role in determining \( V^*_G \) and \( v^{O*}(V) \). We can characterize the effect of a change in the value of \( \beta \) on these variables:

**Corollary 1.** There is a unique \( \beta^* \) so that for values of \( \beta < \beta^* \), \( V^*_G \) declines with \( \beta \), and for \( \beta > \beta^* \), \( V^*_G \) increases with \( \beta \).

The value of the opportunity to grow internally, \( v^{O*}(V) \), declines with \( \beta \).

**Proof.** See Appendix.

First, consider the relation between \( \beta \) and \( V^*_G \). An increase in \( \beta \) reduces the value of internal growth by making less likely that the second stage of the investment will be executed. The firm can improve the probability by reducing the level of \( V^*_G \). The higher \( \beta \) is, the more the firm reduces \( V^*_G \) to protect the value of its subsequent investment. But if \( \beta \) is very high, the probability of the second investment is so low that to increase it requires reducing \( V^*_G \) so much that the value of the first investment is significantly reduced. In this case, \( V^*_G \) increases as \( \beta \) increases.

The relation between \( \beta \) and \( v^{O*}(V) \), however, is monotonic. Recall that the higher \( \beta \) is, the longer the firm needs to wait to proceed with the second stage on average (however the firm chooses \( V^*_G \)). Therefore, \( v^{O*}(V) \) declines with \( \beta \). In the worst case scenario when \( \beta \to \infty \), the firm needs to wait indefinitely, the value of the second-stage investment is zero, and \( v^{O*}(V) = \theta(V^*_G - k)^{\gamma} \beta^{V^*_G - k} \gamma \), the value of the first-stage investment.
2.5 EQUILIBRIUM

In the equilibrium in which the firm chooses to grow by acquisition, even though internal investment does not occur, its value and the exercise strategies for both the first- and second-stage investment are linked to the price that the acquirer pays for the set of assets, because internal investment is a fall-back option if the negotiation does not go through. Given the analytical solution for the value of internal growth in Lemma 1, the equilibrium outcome of the acquisition negotiations, which is given by a pair $p^A(V^A_G)$ and $V^A_G$, is stated in Proposition 1:

**Proposition 1.** The equilibrium levels of $p^A(V^A_G)$ and $V^A_G$ are divided into two regimes according to the ratios $\frac{F}{1-a}$ and $\frac{a + \beta - \gamma (1 - \theta)}{\beta + \gamma (1 - \theta)} k^O$.

**Regime 1 (early acquisition):** For $\frac{F}{1-a} \leq \frac{a + \beta - \gamma (1 - \theta)}{\beta + \gamma (1 - \theta)} k^O$,

$$p^A (V^A_G) = aV^A_G + \rho (V^A_G - F - aV^A_G) - \rho \left( \theta \left( V^O_G - k^O \right) \left( \frac{V^A_G}{V^O_G} \right)^\gamma \right) + (1 - \theta) \left( \beta V^O_G - k^O \right) \left( \frac{V^A_G}{B V^O_G} \right)^\gamma,$$

and

$$V^A_G = \frac{\gamma F}{\gamma - 1 - a}.$$ (9)

**Regime 2 (late acquisition):** For $\frac{F}{1-a} > \frac{a + \beta - \gamma (1 - \theta)}{\beta + \gamma (1 - \theta)} k^O$,

$$p^A (V^A_G) = p^A (V^A_G) = aV^A_G + \rho (V^A_G - F - aV^A_G) - \rho \left( \theta \left( V^A_G - k^O \right) \left( \frac{V^A_G}{V^O_G} \right)^\gamma \right) + (1 - \theta) \left( \beta V^A_G - k^O \right) \left( \frac{1}{\beta} \right)^\gamma,$$

and

$$V^A_G = \frac{\gamma}{\gamma - 1} \frac{\rho (\theta + \beta - \gamma (1 - \theta)) k^O + (1 - \rho) F}{\rho (\theta + \beta - \gamma (1 - \theta)) + (1 - \rho) (1 - a)}.$$ (11)

**Proof.** See Appendix. ■

In the early acquisition regime, the ratio of fixed integration cost, $F$, to the proportion of value added by the acquisition, $1 - a$, is relatively low. In this case, the acquisition threshold, $V^A_G$, is lower than the optimal threshold for internal investment, $V^O_G$. Notice that although $p^A(V^A_G)$ is a function of $\rho$, $V^A_G$ is not. To understand this, recall that the fixed cost the acquirer has to pay under an acquisition
agreement plays an important role for the threshold. This is a deadweight cost for the acquirer, that makes the acquisition less profitable. Thus, the firm is willing to wait for the optimal level of $V$ to acquire the assets.

In the second regime, which we refer to as late acquisition, and which occurs for sufficiently high $\frac{F}{1-a}$, the firm acquires the assets at above the optimal threshold for investing internally, i.e., $V^A_G > V^O_G$. This means that the value of the firm’s outside option is not at the maximum when the firm acquires the assets. Acquiring the assets late reduces the value of the acquirer’s option to grow internally, but the relatively high cost of integrating the acquired assets renders it optimal to do so.

2.6 ACQUISITION AS THE OPTIMAL INVESTMENT STRATEGY

Even though the value increase from an acquisition, $(1-a)\bar{V}$, is positive, an acquisition does not always occur in the equilibrium because internal growth may be more valuable than an acquisition. A condition for an acquisition to be the optimal growth strategy is given in Proposition 2:

**Proposition 2.** When $1 - a > \bar{A} \equiv \left( \frac{(0+\beta^{-\gamma}(1-\theta))F(\gamma-1)+(0+\beta^{-\gamma}(1-\theta))k^O}{0+\beta^{-\gamma}(1-\theta))k^O} \right)$, it is optimal for the firm to acquire the assets rather than to grow internally. $\bar{A}$ increases with $F$, and if $F$ is not too large, $\bar{A}$ declines with $\beta$, and increases with $\sigma$.

**Proof.** See Appendix.

The decision to make an acquisition depends critically on the value created by these assets, $(1-a)\bar{V}$. $\bar{A}$ is the cut-off point above which the firm pursues an acquisition. Above this level, the profits from an acquisition are large enough for the acquirer to prefer to deal with the seller.

Proposition 2 also shows that the cut-off point declines with $\beta$ because as the time necessary to complete the internal expansion increases, the internal growth becomes less attractive. However, when the integration costs, $F$, increase, the acquisition is less profitable; therefore, the cut-off point increases.

Less obvious is the effect of the uncertainty of the underlying assets, $\sigma$, on the cutoff level. The effect of $\sigma$ on the cut-off level is similar to that of $\gamma$. Because both the internal growth and the acquisition are real options, an increase in the uncertainty of the underlying asset increases the value of both options. However, the increase in the value of the internal growth is higher than the increase in the value of the acquisition. To see this, recall that for the internal growth, $\beta$ is a constraint on the second-stage investment, i.e., the second investment will not occur until $V$ reaches $\beta V^O_G$. The increase in the volatility of the underlying assets increases the probability that $\beta V^O_G$ will be reached, and thus helps reduce the adverse effect of
\( \beta \) on the value of the internal growth, so internal growth becomes a more attractive strategy, and, therefore, the cutoff level increases with \( \sigma \).

As we are interested in firms’ acquisition strategies, we focus henceforth on parameters for which an acquisition is the optimal investment strategy.

### 2.7 Acquisition Strategy: Individual vs. Social Efficiency

The social optimum is reached if the outcome of the negotiations is independent of strategic considerations. This takes place when the buyer has full bargaining power, \( \rho = 0 \). In this case, the level at which the acquisition occurs is given in both the early and the late acquisition regime by \( V^*_G = \frac{F}{\gamma - 1 - \alpha} \). As the acquirer does not invest internally, the internal project’s characteristics do not enter the socially efficient threshold.

In the early acquisition regime, the individually and socially efficient thresholds coincide for any distribution of bargaining power, \( \rho \in [0, 1] \). In the late acquisition regime, the acquisition threshold is

\[
V^*_G = \frac{\gamma}{\gamma - 1} \left( 0 + \beta \right) + \left( 1 - \alpha \right) F \rho \left( \theta + \beta \right) + \left( 1 - \rho \right) \left( 1 - \rho \right) F \rho \left( \theta + \beta \right) + \left( 1 - \rho \right) a.
\]

As \( \frac{\partial V^*_G}{\partial \rho} < 0 \) (for a proof see Appendix), the individually optimal threshold is below the socially optimal threshold for \( \rho \in (0, 1] \).

The optimal acquisition threshold for different distributions of bargaining power under the late acquisition regime is displayed in Figure 1, and the base case parameters are identified in Table I. The socially efficient acquisition threshold is \( V^*_G = 2.815 \), which takes place when \( \rho \) is equal to 0. For \( \rho > 0 \), the acquisition is initiated earlier, and when \( \rho = 1 \), it occurs at the internal investment threshold, \( V^*_O \). The value of the acquisition for the acquirer, \( v^*_A \), declines with \( \rho \).

The inability to reach the surplus-maximizing outcome results from the assumptions that an acquisition cannot be agreed upon ex ante and that negotiations take place only once. The acquirer’s outside option deteriorates as the value of the assets moves away from the exercise threshold for internal growth, \( V^*_O \). This strategic element cannot be avoided given the above mentioned assumptions. If, for example, multiple negotiations were allowed, this effect could be eliminated. The inefficiency increases as the seller has more bargaining power.

As a result, policies that limit the bargaining power of the seller may be beneficial to society since they allow the buyer to choose the socially optimal timing of the acquisition. For example, a takeover defense mechanism like a poison pill, which gives the board of directors of the target firm flexibility to negotiate a better deal with the acquirer, may force the acquirer to start the acquisition too early in order to protect the value of its outside option, and, hence, destroy part of the social surplus. From this point of view, policies that curb the use of poison pills may yield more socially desirable outcomes.

Furthermore, acquirer’s strategies that can align the interest of the acquirer and the seller may also bring about a more socially desirable outcome. For example,
Figure 1. The values of acquisition and internal growth (Late acquisition regime). Figure 1 shows the value of the acquisition, \( v^A \), and the value of the internal growth, \( v^O \), for different values of \( \rho \) under the late acquisition regime. A higher \( \rho \) indicates lower bargaining power of the acquirer. The black squares indicate the optimal levels of acquisition thresholds. The parameters are identified in Table I.

Table I. Base Case Parameters

Unless noted, the following parameter configuration is used in all numerical examples.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synergies from acquisition, ( (1 - a) )</td>
<td>0.95</td>
</tr>
<tr>
<td>Payout rate, ( h )</td>
<td>0.15</td>
</tr>
<tr>
<td>Risk-free rate, ( r )</td>
<td>0.05</td>
</tr>
<tr>
<td>Drift rate, ( \mu )</td>
<td>0.25</td>
</tr>
<tr>
<td>Volatility rate, ( \sigma )</td>
<td>0.10</td>
</tr>
<tr>
<td>Initial asset value, ( V )</td>
<td>1.00</td>
</tr>
<tr>
<td>Total internal investment, ( k^O )</td>
<td>1.00</td>
</tr>
<tr>
<td>Fixed Integration cost, ( F )</td>
<td>0.85</td>
</tr>
<tr>
<td>First-stage investment, ( \theta )</td>
<td>0.50</td>
</tr>
<tr>
<td>Second-stage delay, ( \beta )</td>
<td>1.50</td>
</tr>
</tbody>
</table>

when the acquirer possesses a stake or “toehold” in a takeover target, it will start the acquisition later than it otherwise would in order to increase the value of its stake in the target firm. Therefore, the acquisition will take place closer to a socially optimal threshold.

The result that the acquisition may not be initiated at the level that maximizes the overall surplus contrasts with the result in Morellec and Zhdanov (2005) and
Lambrecht (2004) who obtain a socially efficient outcome. The inefficiency in our model occurs because of the assumptions that the buyer has the internal growth alternative, and negotiations take place once, and it is not possible to strike a deal ex ante, which is not the case in Morellec and Zhdanov (2005) and Lambrecht (2004).

3. Profitability of Internal Growth and Acquisition Strategy

The equilibrium $V_A^*$ and $v_A^*(V)$ change with the main parameters that characterize internal growth. Proposition 3 summarizes the effects of changes of $\beta$ on $V_A^*$ and $v_A^*(V)$ for any exogenous value of $\rho$:

**Proposition 3.** In the case of an early acquisition, $\beta$ does not affect $V_A^*$. In the case of a late acquisition and for any value of $0 < \rho \leq 1$, there is a unique $\beta^*$ so that for $\beta < \beta^*$, $V_A^*$ declines with $\beta$, and for $\beta > \beta^*$, $V_A^*$ increases with $\beta$. In both cases, the value of the opportunity to acquire, $v_A^*(V)$, declines with $\beta$.

**Proof.** See Appendix. ■

The fact that $v_A^*(V)$ declines with $\beta$ in the late acquisition case is straightforward. As internal growth is the acquirer’s outside option, when it is less valuable, so is the acquisition. How much the acquisition’s value is reduced depends on the bargaining power of the acquirer. If the seller has all the bargaining power, the values of both the acquisition and the internal growth decline by the same amount.

Next, consider the relation between $\beta$ and $V_A^*$. Recall from Corollary 1 that for $\beta < \beta^*$, the higher the $\beta$, the lower the $V_O^*$, and that the opposite holds for $\beta > \beta^*$. The relation between $\beta$ and $V_A^*$ is similar, but the amount of change in $V_A^*$ depends on the bargaining power. If the seller has all the bargaining power, then $V_A^* = V_O^*$, and the change in $V_A^*$ is equal to the change in $V_O^*$.

Figure 2 shows how equilibrium $p^A(V_A^*)$ and $V_A^*$ vary with $\beta$, and how the acquisition value and the seller’s wealth are affected by $\beta$ under the parameters set forth in Table 1.

In Panel A, $V_A^*$ declines with $\beta$ until $\beta = 4.24$, and then increases slightly thereafter. Panel B shows that $p^A(V_A^*)$ under the current parameter configuration increases with $\beta$. The higher $\beta$ is, the less valuable internal growth is, and the higher the price the seller can charge the acquirer. This makes the seller’s wealth increase and the acquisition value decline with $\beta$ as shown in Panel C.

4. Imperfect Information about the Acquirer’s Outside Option

We would like to provide an explanation for observed price patterns before and around acquisition announcements. To do this, we assume investors have imperfect
Figure 2. Effects of a delay on the second-stage investment (Late acquisition regime). Panel A shows $V_G^A$ as a function of $\beta$; Panel B shows $p^A$ as a function of $\beta$; Panel C shows the acquisition value, $v^A$ (solid line), and the seller’s wealth, $s$ (dashed line), as functions of $\beta$. The seller is assumed to have all bargaining power, i.e., $\rho = 1$, and the acquisition occurs relatively late (the late acquisition regime). The other parameters are identified in Table I.
information about the expected time to complete the internal investment, \( \beta \). If the managers of the acquiring firm observe the true value of \( \beta \), but either cannot communicate or choose not to communicate it to outside investors, the acquisition announcement may resolve this uncertainty for investors.\(^9\)

We examine the learning effect of the acquisition announcement, assuming \( \rho = 1 \). A different distribution of bargaining power does not affect the direction of the results. Managers of the acquirer are assumed to maximize the intrinsic value of the firm, which rules out preferential treatment of current shareholders over future shareholders. Also, we assume that the seller learns the true value of \( \beta \) during the course of the negotiations.\(^10\)

Investors’ prior distribution of \( \beta \) at time \( t = 0 \) is denoted by \( F(\beta) \) with density \( f(\beta) \). We assume that for all \( \beta \in [\beta, \beta^*] \), an acquisition creates a positive surplus. Assume for the moment that \( \beta \geq 1 \) and \( \beta \leq \beta^* \). The information set of outside investors at time \( t \) is \( \mathcal{F}_t \), which includes the history of \( V \) from time \( t = 0 \) to time \( t \) and the value of \( V = V^{A*}_G \) at which the firm acquires the target.

4.1 THE EFFECT OF AN ACQUISITION ANNOUNCEMENT ON THE STOCK PRICE

Let \( \hat{v}^A_t \) be the outside investors’ expected value at time \( t \) of the acquiring firm. Then:

\[
\hat{v}^A_t = E[v^{A*}(V)|\mathcal{F}_t],
\]

where for notational convenience we suppress the dependence of \( v^{A*}(V) \) on \( V^{A*}_G \), where \( V^{A*}_G \) is in turn a function of \( \beta \). If we denote the information set that includes everything up to time \( t \) but excludes the acquisition announcement by \( \mathcal{F}'_t \), then:

\[
\hat{v}^{A'}_t = E[v^{A*}(V)|\mathcal{F}'_t]
\]

is the expectation of \( v^{A*}(V) \) when investors do not observe \( V^{A*}_G(\beta) \), but observe the development of \( V \).

\(^9\) That option exercise can convey information is explicitly analyzed in Grenadier (1999), Lambrecht and Perraudin (2003), Carlson et al. (2004), and Morelec and Zhdanov (2005).

\(^10\) While we treat the distribution of bargaining power as exogenous, certain differences in bargaining power appear to be more common than others. Bargaining power in negotiations is determined by the patience of the bargaining parties (see, for example, Osborne and Rubinstein (1990), Chapters 3 and 4). While the seller can operate efficiently during negotiations, the acquiring firm typically must commit resources to organize the planned integration and may thus forego other opportunities if negotiations take too long. It is then natural to assume that the acquiring firm is less patient than the seller and therefore has less bargaining power. Also, if the ownership of the seller is dispersed, the free-rider problem makes the seller’s owners more patient (Grossman and Hart, 1980).
We define the announcement effect as a change in the value of the acquisition at the time of its announcement, $\tau$, as:

$$\Delta^A_t = \tilde{v}^A_t - \tilde{v}^A_{t+}. \quad (15)$$

Even if investors observe the announcement of an acquisition, they can only draw inferences about the realization of $\beta$, $\beta^\#$, when $V^G_A$ is a function of $\beta$. Recall from (12) that when $\frac{F}{1-a} \leq \frac{\theta+\beta\gamma}{\theta+\beta^\gamma}(1-\theta) k \theta$, $V^G_A = \frac{\gamma}{\gamma-1} \frac{F}{1-a}$. In this case, investors are able to perfectly anticipate the acquisition threshold, as $V^G_A$ is independent of $\beta$, and no information is transmitted by the acquisition announcement. Therefore, the posterior density $f_t(\beta)$ at any time $t > 0$ is the same as the prior, i.e., $f_t(\beta) = f(\beta)$. The expectation of the value of an acquisition is:

$$\tilde{v}^A_t = E[V^A_t(V)]. \quad (16)$$

where the expectation is calculated using the prior density.

When $\frac{F}{1-a} > \frac{\theta+\beta\gamma}{\theta+\beta^\gamma}(1-\theta) k \theta$, $V^G_A$ varies with $\beta$. For the assumed support, the function $V^G_A(\beta)$ declines monotonically with $\beta$. Thus, any value of $V^G_A$ corresponds to a unique value of $\beta$, implying that outside investors can estimate the realized value of $\beta$ perfectly upon observing $V^G_A$ in the form of an acquisition announcement.

Before the acquisition, investors revise their beliefs regarding the realization of $\beta$. If no acquisition is announced, investors know the acquisition threshold must be higher than the highest value of $V$ up to that time, $V^G_A > V^{\text{mt}} = \sup_{t \leq \tau} V_t$. Thus, every time $V$ reaches a new peak, information is conveyed. Since from Proposition 3 for $\beta < \beta^\#$, $V^G_A(\beta)$ declines with $\beta$, there is an inverse function $G^{-1}$ such that $G^{-1}[V^G_A(\beta)] = \beta^\#$. Because of the inverse relation between $V^G_A$ and $\beta^\#$, $\frac{dG^{-1}}{d\beta} < 0$. As the acquisition has not occurred yet, the true value of $\beta$ must be lower than $G^{-1}(V^{\text{mt}})$. Given this information, investors update their beliefs by conditioning that $\beta < G^{-1}(V^{\text{mt}})$, so the posterior density at any time $t < \tau$, $f_t(\beta)$, is:

$$f_t(\beta) = \Pr(\beta | \beta < G^{-1}(V^{\text{mt}})) = \frac{f(\beta)}{F(G^{-1}(V^{\text{mt}}))}. \quad (17)$$

For $t \geq \tau$, investors observe $V^G_A(\beta)$ and therefore infer the true value of $\beta$ perfectly. Thus, the expectation of the value of an acquisition is:

$$\tilde{v}^A_t = \begin{cases} E[V^A_t(V) | V^G_A(\beta) > V^{\text{mt}}], & \text{when } t \in [0, \tau) \, , \\ v^A_t(V), & \text{when } t \in [\tau, \infty) \, , \end{cases} \quad (18)$$

calculated using the posterior density at time $t$, and $v^A_t(V)$ at $t \geq \tau$ is the value of $v^A_t(V)$ evaluated at $\beta^\#$. Proposition 4 summarizes the effect of an acquisition announcement on the acquisition value:
Proposition 4. If the seller has all the bargaining power, and internal growth is not expected to take too long to complete, \( \bar{\beta} \leq \beta^* \), it holds that if \( \frac{F}{1-a} \leq \frac{\theta + \bar{\beta} \gamma (1-\theta)}{\theta + \bar{\beta} \gamma (1-\theta)} k^O, \Delta^A = 0 \), and for any \( \beta^* \in \left[ \bar{\beta}, \bar{\beta} \right] \) if \( \frac{F}{1-a} > \frac{\theta + \bar{\beta} \gamma (1-\theta)}{\theta + \bar{\beta} \gamma (1-\theta)} k^O, \Delta^A < 0 \).

Proof. See Appendix.

When \( \frac{F}{1-a} < \frac{\theta + \bar{\beta} \gamma (1-\theta)}{\theta + \bar{\beta} \gamma (1-\theta)} k^O \), the acquisition occurs at a relatively low threshold, and \( V^G_A < V^G_0 \). In this case, the acquisition threshold does not depend on \( \beta \), and the acquisition itself does not convey any information about it. Only the price of the acquisition reveals \( \beta^* \). If expectations of \( \beta \) have been formed rationally, however, the announcement effect is zero on average.

When \( \frac{F}{1-a} \geq \frac{\theta + \bar{\beta} \gamma (1-\theta)}{\theta + \bar{\beta} \gamma (1-\theta)} k^O \), and \( V \) reaches a new peak without there being an acquisition, investors conclude that the value of \( \beta \) that corresponds to the current \( V \) as the acquisition threshold is not \( \beta^* \). This is the highest possible value of \( \beta \) in the distribution. There is still uncertainty about the lower values of \( \beta \). Consequently, investors update their distribution of \( \beta \) by lowering its upper bound, which reduces the expected \( \beta \). The acquisition announcement resolves any uncertainty about these possible lower values and reveals \( \beta^* \). Therefore, \( \beta^* \) is always higher than the expected \( \beta \) and \( \Delta^A > \Delta^A \). The acquisition announcement is bad news for the investor, and the model generates negative announcement returns of acquisitions. Other studies that report negative announcement effects are McCardle and Viswanathan (1994) and Jovanovic and Braguinsky (2004); however their results are based on uncertainty about whether an acquisition will occur, rather than when an acquisition takes place. The results are also consistent with empirical evidence (see Schwert, 2000, and Andrade et al., 2001).

These results hold when the expected second-stage delay, \( \beta \), is not too long, \( \bar{\beta} < \beta^* \). When there is a long delay, \( \beta > \beta^* \), the results are reversed. In this case, an acquisition is good news, as an increase in \( \beta \) implies a higher acquisition threshold. Longer inactivity by the acquirer is interpreted as a negative signal as to the prospects of internal growth and has a negative impact on its stock price.

When \( \beta \leq \beta^* \leq \bar{\beta} \), the function \( V^G_A(\beta) \) does not monotonically decline with \( \beta \). As Proposition 3 indicates, \( V^G_A(\beta) \) declines with \( \beta \) for \( \beta < \beta^* \), and increases with \( \beta \) for \( \beta \geq \beta^* \). In this case, investors may not always perfectly anticipate \( \beta^* \) from the announcement of the acquisition, and the effect of the acquisition announcement is

11 The financial press appears to believe that negative announcement returns are not always an indicator of overpayment. For example, an article on Procter & Gamble’s acquisition of Gillette reads: “In midday trading yesterday, P&G shares were trading at $53.87, down 2.6 per cent. To many in the mergers and acquisitions world, such a small drop in the acquirer’s share price ... amounted to Wall Street’s endorsement of the structure of the deal” [Financial Times, January 29 and 30, 2005].
dependent on the prior distribution of $\beta$. However, for reasonable parameter values, the upper bound of $\beta$ is likely to be lower than $\beta^*$, i.e., for $(\mu - \delta) = 0.2$, $\sigma = 0.10$, if $\beta = \beta^* = 4.24$, it will take on average as long as 7.1 years to complete the internal growth, so it is not unreasonable to assume that $\beta < \beta^* = 4.24$.

4.2 DISCUSSION: ROBUSTNESS OF THE RESULTS

The acquirer benefits from a large outside option at the time of the acquisition, which influences the optimal acquisition threshold. The outside option takes the form of the value of the acquirer’s internal investment opportunity. Thus, as the optimal threshold varies as a function of a change in one of its characteristics, the optimal acquisition threshold either remains constant (early acquisition) or moves in the same direction (late acquisition). The critical property that drives negative announcement returns in the model is that the optimal threshold for investing internally rises with the profitability of the investment. In other words, the acquirer can wait longer to acquire, because internal investment is more profitable. As long as this property is preserved, alternative model formulations yield the same result.

One example of an alternative formulation is that the profitability of investing internally is characterized not by a given time-to-build but by a probability that the investment will evaporate at any given time. Then, a more profitable internal investment is one that has a lower probability of disappearing. In this case, a firm with a more profitable investment has a higher optimal investment threshold, as the probability of losing the investment is relatively low. Such a model features a positive relation between profitability and investment threshold, and can be used to explain negative announcement returns.

Under our framework, the model generates negative stock price reactions to acquisition announcements in the late acquisition regime and negligible expected returns otherwise, not just when investors are uncertain about the expected time to complete the internal investment, but also when they are uncertain about the distribution of bargaining power between buyer and seller. As the seller has greater bargaining power, $\rho$, it becomes more important for the acquirer to initiate the acquisition when its outside option is more valuable. As a consequence, the optimal acquisition threshold declines monotonically with $\rho$. When investors update their

12 If $\beta$ is not too high, i.e., $V_{G^A}^*(\beta) > V_{G^A}^{\hat{\beta}}(\beta)$, even though $\beta \leq \beta^* \leq \beta$, there always exists $\hat{\beta} < \beta$ such that $V_{G^A}^{\hat{\beta}}(\beta) = V_{G^A}^*(\beta)$, and for any $\beta^* \in [\hat{\beta}, \beta)$, $\Delta_1^A < 0$. This means that if the upper bound of the distribution of $\beta$ is not too high, there exists a range of $\beta$ such that $V_{G^A}(\beta)$ monotonically declines with $\beta$ and the investors can perfectly anticipate $\beta^*$. In this range, the negative announcement effect occurs independently of the distribution of $\beta$. Also, note that in the special case in which $\theta = 0$, $V_{G^A}^*(\beta)$ strictly declines with $\beta$, and the negative announcement effect occurs regardless of the upper bound and the distribution of $\beta$.

13 Such a model is analyzed in McDonald and Siegel (1986) and Carlson et al. (2004).
beliefs regarding the distribution of bargaining power rationally as described above, announcement of an acquisition is negative news, because it reveals that the acquirer has less than expected bargaining power.

4.3 STOCK PERFORMANCE PRIOR TO THE ACQUISITION ANNOUNCEMENT

The updating of $\beta$ also creates a steeper price run-up in the value of an acquisition than in the case of no learning by investors. The price run-up in the value of the acquirer occurs for two reasons. First, in order for an acquisition to occur, $V$ must move up and reach the acquisition threshold. Second, as $V$ approaches the threshold, it reaches new peaks, and investors revise their beliefs and raise their expectation of the profitability of internal investment.

To see the effect of the updating before the announcement consider two times, $t_1 < t_2 < \tau$, when the sets of assets are equally valuable. Define the change in the expected value of the acquisition from the perspective of outside investors from time $t_1$ to $t_2$ as:

$$\Delta_{t_1,t_2}^A = \hat{v}_{t_2}^A - \hat{v}_{t_1}^A.$$  \hspace{1cm} (19)

Proposition 5 characterizes $\Delta_{t_1,t_2}^A$ before the time of the announcement:

**Proposition 5.** Suppose the seller has all the bargaining power, and it is not expected to take too long to complete internal growth, $\bar{\beta} \leq \beta^*$. If at two times, $t_1$ and $t_2$, the value of the assets, $V$, is the same, then for $t_1 < t_2 < \tau$, $\Delta_{t_1,t_2}^A \geq 0$. If $V^{m_{t_1}} < V^{m_{t_2}}$ and $G^{-1}(V^{m_{t_1}}) > G^{-1}(V^{m_{t_2}})$, then $\Delta_{t_1,t_2}^A > 0$.

**Proof.** See Appendix. \hfill \blacksquare

Without any revision in expectation, we would have $\Delta_{t_1,t_2}^A = 0$, because the assets have the same value at both dates. Investors update their expectation of $\beta$ as time passes. As $V$ reaches a new peak between time $t_1$ and $t_2$, investors form a posterior distribution by eliminating the highest $\beta$ thought possible before the peak was reached. Thus, for any $t_2 > t_1$, $E[\beta]$ at $t_2 \leq E[\beta]$ at $t_1$, and $\hat{v}_{t_2}^A \geq \hat{v}_{t_1}^A$. If between $t_1$ and $t_2$, $V$ reaches a new high ($V^{m_{t_1}} < V^{m_{t_2}}$), values are strictly unequal, as investors are revising the distribution of $\beta$. In this sense, the model generates greater positive changes in the acquisition value than the changes generated by the evolution of the value of the underlying asset alone.

4.4 NUMERICAL EXAMPLE

A numerical example demonstrates how investors update their beliefs about $\beta$, and how the announcement effect is generated. We assume that $\beta^* = 1.5$, and investors’
Figure 3. Announcement effects (Late acquisition regime). Panel A shows evolution of the value of the underlying asset, $V$ (solid line), and the maximum value of $V$ up to time $t$, $V^{mt}$ (dashed line). Panel B shows the expected value of $\hat{\beta}$ that corresponds to $V^{mt}$. Panel C shows investors’ expected value of an acquisition, $E[v^A]$ (solid line), and the value of an acquisition when the true value of $\hat{\beta}$ is known, $v^A$ (dashed line). Panel D plots the percentage difference between the expected acquisition value, $E[v^A]$, and the acquisition value if $\hat{\beta}$ is known, $v^A$. The distribution of $\hat{\beta}$ is assumed to be uniform over $[1, 4]$, and the true $\beta^*$ is 1.5. The initial asset value $V$ is 2.5 and $\rho = 1$. The other parameters are identified in Table I.

prior distribution of $\beta$ is uniform over 1 and $4 < \beta^*$. The other parameters are as identified in Table I. Given the parameters, the time lag between the first and second investment will take on average 2.07 years. Figure 3 shows the development of $V$, $V^{mt}$, $E[\hat{\beta}]$, and $v^A$ before and after the announcement.

Panel A shows the evolutions of $V$ (solid line) and $V^{mt}$ (dashed line). At time $t = 0$, $V$ is equal to 2.50. It reaches the acquisition threshold at time $\tau = 50$ at $V^A_\text{G}(\beta) = 2.67$. Recall that by definition $V^A_\text{G}(\beta) = V^{mt}$; that is, the acquisition threshold is the highest value of $V$ up to time $\tau$.

The corresponding expected $\hat{\beta}$ is shown in Panel B. An unconditional expectation of $\hat{\beta}$ is equal to $2.5 \left( \frac{1}{2} (1 + 4) \right)$. At $t = 0$, no announcement has occurred, so investors deduce that $V^A_\text{G}(\hat{\beta}) > 2.5$, and $\beta < 1.67$. This leads them to form an expectation of $\beta$ that is equal to $1.33 \left( \frac{1}{2} (1 + 1.67) \right)$.
As $V$ reaches higher values, investors start revising their expectation of $\beta$. For example, at $t = 20$, $V$ reaches 2.6, and no announcement has occurred, so investors deduce that $V^A_G(\beta) > 2.6$, and $\beta < 1.65$. This leads them to form an expectation of $\beta$ that is equal to 1.325 ($\frac{2}{7} (1 + 1.65)$). Between $t = 20$ and $t = 42$, $V$ does not reach any new peak, so there is no revision of the expected value of $\beta$ in this period.

At $\tau = 50$, the acquisition is announced, and investors learn that $V^A_G(\beta) = 2.67$, and conclude that $\beta^h = 1.5$. Therefore, there is a positive jump in the value of expected $\beta$ at this point.

This jump corresponds to a decline at time $\tau = 50$ in the expected value of the acquisition, $\hat{v}^A$, represented by a solid line in Panel C. For comparison, we also include in Panel C the acquisition value if $\beta$ is known from time $t = 0$ (represented by the dashed line). Notice that as $t$ approaches $\tau = 50$, the investors' expectation of $\beta$ is lower than $\beta^h$, therefore the expected acquisition value is higher than the true value. At $\tau$, uncertainty about $\beta$ is resolved, and the lines follow identical paths from this time forward.

This effect can be clearly seen in Panel D, which plots the percentage difference between the expected acquisition value and the acquisition value if $\beta$ is known. Before the acquisition, because investors revise their expectation of $\beta$, there is a greater difference as $V$ reaches new peaks. When there is no learning about $\beta$, the difference may grow or shrink with $V$. Overall, these changes are dominated by the effect of the learning about $\beta$. Thus, there is a greater difference between the expected acquisition value and the acquisition over time before the acquisition and the difference disappears when the acquisition is announced.

### 4.5 EXTENSION: EITHER PARTY CAN INITIATE THE TRANSACTION

So far, we have assumed that the buyer initiates the negotiations. Extending the analysis to allow for the initiation of negotiations by either party both provides a robustness check of the results and yields insights about which transactions are initiated by the seller and which by the buyer.

We maintain the assumption that negotiations take place only once. If negotiations fail, each party is left with its outside options.

**Lemma 2.** In the early acquisition regime, the seller’s utility maximizing threshold, $V^S_G = \frac{\gamma}{\gamma - 1} \frac{F}{1 - a}$. In the late acquisition regime, $V^S_G \geq \frac{\gamma}{\gamma - 1} \frac{F}{1 - a}$.

**Proof.** See Appendix.

In the early acquisition regime, the seller does not benefit from waiting beyond the surplus maximizing value, so $V^S_G = V^A_G = \frac{\gamma}{\gamma - 1} \frac{F}{1 - a}$. This is because the integration cost, $F$, is small. If the synergy created by acquisition is sufficiently large,
the seller’s surplus declines significantly if it waits beyond $V_{G}^{A*}$. Although, the acquisition price for the seller improves slightly if it waits beyond $V_{G}^{O*}$ because the value of the buyer’s internal growth option starts to deteriorate, the increase is not enough to compensate for the decline in the seller’s surplus.

In the late acquisition regime, the seller’s optimal threshold, $V_{S}^{*}$, is at least as high as the the surplus maximizing threshold, i.e., $V_{G}^{S*} \geq \frac{\gamma}{\gamma-1} \frac{F}{1-F}$. This occurs because a higher threshold implies a less valuable option for the buyer, and, consequently, a higher acquisition price for the seller, and in this case $F$ is relatively large; therefore, the loss in the seller’s share of social surplus is small compared to the increase in price for the seller.

Lemma 2 is an important ingredient in establishing the identity of the party that initiates negotiations. The proposition is:

**Proposition 6.** If the buyer and the seller can initiate the acquisition negotiations, in the early acquisition regime, the negotiations may be initiated by either the buyer or the seller. In the late acquisition regime, the negotiations are always initiated by the buyer.

**Proof.** See Appendix.

In the early acquisition regime, the surplus maximizing threshold lies above the optimal threshold for starting the internal investment. As in this case, a marginal increase in the threshold does not affect the buyer’s outside option, and, similar to Lambrecht (2004) and Morellec and Zhdanov (2005), the seller does not have the incentive to delay the negotiation beyond the socially optimal level. Then, both parties choose the identical threshold of $\frac{\gamma}{\gamma-1} \frac{F}{1-F}$, and it is indeterminate who initiates the negotiations.

In the late acquisition regime, the buyer’s optimal threshold is below the surplus maximizing level. Then, the seller never finds it optimal to preempt the buyer’s initiation of the negotiations, since its utility increases in the acquisition threshold. So the buyer initiates the negotiation at $V_{G}^{A*} < V_{G}^{S*}$. Although, the seller can get a higher utility if it can delay the negotiation, anticipating that the negotiation will take place only once, it will rationally accept the initial offer. This result is identical to the equilibrium outcomes in Section 2.

### 5. Empirical and Policy Implications

Our model supports a number of empirical and policy implications. As it relates acquisition decisions to the characteristics of internal investment opportunities, it is most useful for situations in which internal investment is a realistic alternative to
an acquisition. It should also be noted that the following implications do not rely on any agency consideration.

**Acquisition Initiation**

The model demonstrates that the decision to initiate acquisition negotiations is made strategically by the acquirer and the seller. While acquisition thresholds are identical for both parties at ratios of integration cost to synergy, \( \frac{F}{1-a} \), below a certain level, above this level the seller chooses a higher valuation than the buyer. Thus, transactions with high \( \frac{F}{1-a} \) will be initiated by buyers, while those with a low \( \frac{F}{1-a} \) may be initiated by either the buyer or the seller.

**Announcement Returns, Integration Costs, and Synergies**

Announcement returns for the acquirer depend critically on the relative integration cost and proportional value added. For low values of \( \frac{F}{1-a} \) announcement returns are zero, and for high values announcement returns are in general negative. Thus, acquisitions providing fewer synergies relative to integration costs are predicted to result in lower acquirer announcement returns.

Our model is the first to predict a relation between announcement returns and the levels of integration cost and the synergies from an acquisition, without requiring learning about these variables. Our result is driven by investor uncertainty about either the profitability of the internal growth alternative or the relative bargaining power of the players. Contrary to results in Morellec and Zhdanov (2005), our result does not depend on competition among different bidders.

Stock price effects of announcements are predicted even though investors correctly anticipate that the firm chooses an acquisition as its method to grow. This finding complements the results in McCardle and Viswanathan (1994) and in Jovanovic and Braguinsky (2004), who report negative announcement effects based on uncertainty about whether an acquisition will occur, rather than when an acquisition takes place.

**Acquisition Initiation and Acquirer Announcement Returns**

As acquisitions with high \( \frac{F}{1-a} \) are initiated by the acquirer, acquirer-initiated transactions are predicted to generate negative announcement effects on average, while returns in seller-initiated transactions are expected to be negligible.

**Pre-announcement Run-up**

In our model, the price run-up before the acquisition occurs because of imperfect information regarding the value of the option to grow, and is consistent with the empirical evidence in Harford (1999) and in Ang and Cheng (2006). In a cross
section analysis, our model predicts greater stock price run-up when \( \frac{F}{P} \) is higher, because of positive updating of the acquirer’s internal investment profitability in these cases. As a relatively high integration cost-to-synergy ratio implies negative announcement returns, the model predicts a negative correlation between the pre-announcement price run-up and announcement returns.

**Bargaining Power and Social Efficiency**

Our work shows that an acquisition is not always initiated at a level that maximizes the overall surplus to society. This is in contrast to Jovanovic and Braginisky (2004), who find that in a competitive industry mergers are not just privately, but also socially efficient.

In the case of high integration cost or low value added, the greater the seller’s bargaining power, the lower the acquisition threshold and the lower the surplus generated by the acquisition. Hence, policies that limit the bargaining power of a seller may be beneficial to society; they help the buyer choose the socially optimal timing of the acquisition. The greater bargaining power in the hands of the seller can force the acquirer to start the acquisition too soon, as a way of protecting its outside option. This destroys social surplus. Consequently, from a social welfare view, policies that restrict the use of poison pills should yield more socially desirable outcomes.

6. Conclusion

We compare a firm’s opportunity to grow internally with the option of expanding via acquisitions. The advantage of an acquisition over internal investment is quicker access to cash flows. The disadvantage is, in general, a higher cost in the price paid for the acquired business, plus any integration expenses.

When there is significant cost to integrate an acquired business, we show that the opportunity to achieve growth via internal investment influences the acquisition strategy. This is because the value of internal growth gives the firm another option in bargaining with the seller. The value of this option is constant up to a certain value of the asset to be acquired, but declines above that value.

When a relatively high acquisition cost leads to a high acquisition threshold, the declining value of the outside option reduces this threshold to a lower asset value. This makes acquisitions occurring sooner than if there were no internal growth opportunities, thus creating social inefficiency. A thorough analysis of the effects of different regulatory policies or acquisition strategies on the social efficiency is an interesting avenue for future research both theoretically and empirically.

For a wide range of parameters, acquisitions occur earlier, the longer the time between initiating and completing internal growth. This implies negative stock price
reactions to buyer-initiated acquisition announcements, and price run-ups prior to acquisitions when investors are imperfectly informed about the profitability of internal investment. Seller-initiated acquisitions lead to negative stock price movements when the seller has considerable bargaining power.

**Appendix**

**Proof of Equation 2.** A value function \( f(V) \), which is dependent on a state variable \( V \) that follows (1), must satisfy the differential equation:

\[
\frac{1}{2} \sigma^2 V f_{VV}(V) + (r - \delta) V f_{V}(V) - rf(V) = 0, \tag{A-1}
\]

The general solution of the ODE (A-1) is

\[
f(V) = X_1 V^\gamma + X_2 V^\gamma', \tag{A-2}
\]

where \( X_1 \) and \( X_2 \) are constants to be determined, and \( \gamma > 1, \gamma' < 0 \) are quadratic roots of the equation:

\[
\frac{1}{2} \sigma^2 x(x - 1) + (r - \delta)x - r = 0. \tag{A-3}
\]

To solve for the value function \( v^A(V) \) for an arbitrary value of \( V_G^A \), we use the boundary conditions:

\[
v^A(0) = 0 \tag{A-4}
\]

\[
v^A(V_G^A) = (V_G^A - k^A(V_G^A)). \tag{A-5}
\]

(A-4) implies that \( X_2 = 0 \), or the value function will reach \( \infty \), when \( V \) approaches 0. Notice that there is no smooth-pasting condition since the investment rule is arbitrarily given by \( V_G^A \). We then solve for the coefficient \( X_1 \) and obtain:

\[
v^A(V) = (V_G^A - k^A(V_G^A)) \left( \frac{V}{V_G^A} \right)^\gamma. \tag{A-6}
\]

**Proof of Equation 6.** The target firm’s value function, \( s(V) \), for an arbitrary value of \( V_G^A \) can be thought of as the current asset value plus an option to obtain \( p^A(V_G^A) \) in exchange of \( aV_G^A \), i.e., \( s(V) = aV + e(V) \), where \( e(V) \) is an option to obtain \( p^A(V_G^A) \) with the cost \( aV_G^A \). We need to find the value function for the option \( e(V) \), which satisfies (A-1), but has different boundary conditions:

\[
e(0) = 0 \tag{A-7}
\]

\[
e(V_G^A) = p^A(V_G^A) - aV_G^A. \tag{A-8}
\]

\[\text{For simplicity, we suppress the dependence of } v^i(V) \text{ on } V_G^A \text{ and write } v^A(V) \text{ instead of } v^i(V, V_G^A).\]
Again there is no smooth-pasting condition since the seller does not choose optimal \( V_G^A \). (A-7) implies that the coefficient \( X_2 = 0 \). Using (A-8) to solve for \( X_1 \) in the general solution gives

\[
e(V) = (p^A(V^A_G) - aV^A_G)(\frac{V}{V_G^A})^\gamma \, .
\] (A-9)

**Proof of Lemma 1.** To obtain the value function \( v^O(V) \), we start by solving the value of the second-stage investment, \( v^O'(V) \), which must satisfy the boundary conditions:

\[
v^O(0) = 0 \quad (A-10)
\]
\[
v^O'(\beta V) = (1 - \theta)(\beta V - k). \quad (A-11)
\]

(A-10) implies that \( X_2 = 0 \), and there is no smooth-pasting condition in this stage since the investment rule is already specified by \( \beta \). We then solve for the coefficient \( X_1 \) and obtain

\[
v^O'(V) = (1 - \theta)(\beta V - k^O) \left(\frac{1}{\beta}\right)^\gamma . \quad (A-12)
\]

Recall that the investment in the first stage gives the firm \( \theta \) of \( V \) and access to the second-stage investment, \( v^O'(V) \). The value function of the first-stage investment, \( v^O(V) \), must satisfy the boundary and smooth-pasting conditions:

\[
v^O(0) = 0 \quad (A-13)
\]
\[
v^O(V^O_G^*) = \theta (V^O_G^* - k^O) + (1 - \theta)(\beta V^O_G^* - k^O) \left(\frac{1}{\beta}\right)^\gamma . \quad (A-14)
\]
\[
\frac{\partial v^O(V)}{\partial V} \bigg|_{V = V^O_G^*} = \theta + (1 - \theta)\beta^{1-\gamma} . \quad (A-15)
\]

As before, \( X_2 = 0 \), and we are left with two unknown variables, \( X_1 \) and \( V^O_G^* \), and two equations. Solving for \( X_1 \) and \( X_2 \) and rearranging yield

\[
v^O^*(V) = \begin{cases} 
\theta(V - k^O) + (1 - \theta)(\beta V - k^O) \left(\frac{1}{\beta}\right)^\gamma & \text{for } V > V^O_G^* \\
\theta(V^O_G^* - k^O) \left(\frac{V}{V_G^*}\right)^\gamma + (1 - \theta)(\beta V^O_G^* - k^O) \left(\frac{V}{\beta V_G^*}\right)^\gamma & \text{for } V \leq V^O_G^* 
\end{cases} \quad (A-16)
\]

and

\[
V^O_G^* = \frac{\gamma}{\gamma - 1} \frac{\theta + \beta^{-\gamma}(1 - \theta)}{\theta + \beta^{1-\gamma}(1 - \theta)} k^O . \quad (A-17)
\]
Proof of Corollary 1. To see that $V^O_G$ is not a monotonic function of $\beta$ and that there is a unique $\beta^*$ such that for $\beta > \beta^*$, $V^O_G$ increases with $\beta$, and for $\beta < \beta^*$, declines with $\beta$, differentiate (8) with respect to $\beta$:

$$\frac{\partial V^O_G}{\partial \beta} = \frac{(1 - \theta)x(\beta)}{\beta(1 + (\beta^{-1} - 1)\theta)^2 \gamma - 1} k^O,$$

(A-18)

where

$$x(\beta) = \beta^\gamma \theta \left[ \frac{\beta - 1}{\beta} - 1 \right] - (1 - \theta).$$

(A-19)

Because $\gamma > 1$, $\beta > 1$, and $\theta < 1$, the numerator of (A-18) can be positive or negative, depending on $x(\beta)$. Next, we will show there is a unique $\beta^*$ such that for values of $\beta < \beta^*$, $x(\beta^*) < 0$, and for $\beta > \beta^*$, $x(\beta^*) \geq 0$, and $\frac{\partial V^O_G}{\partial \beta} \geq 0$.

Differentiating $x(\beta)$ with respect to $\beta$ gives

$$\frac{\partial x(\beta)}{\partial \beta} = \beta^{2+\gamma}(\beta - 1)\theta\gamma(\gamma - 1) > 0,$$

(A-20)

which means that $x(\beta)$ is increasing in $\beta$.

Evaluating $x(\beta)$ at $\beta = 1$, and $\beta \to \infty$ gives $x(1) = -1$, and $x(\infty) = +\infty$. We therefore conclude that there is $\beta^*$ such that $x(\beta^*) = 0$, and $\frac{\partial V^O_G}{\partial \beta} = 0$. For any $\beta < \beta^*$, we have $x(\beta^*) < 0$ and $\frac{\partial V^O_G}{\partial \beta} < 0$. For any $\beta > \beta^*$, we have $x(\beta^*) \geq 0$, and $\frac{\partial V^O_G}{\partial \beta} \geq 0$.

Next, to see that $v^O(A)$ declines with $\beta$, differentiate $v^O(A)$ with respect to $\beta$:

$$\frac{\partial v^O(A)}{\partial \beta} = (\theta - 1)\theta \frac{\beta(1 + (\beta^{-1} - 1)\theta)^{-1} (\gamma - 1)^\gamma}{\gamma - 1} V^y(k^O)^{1-y} < 0,$$

(A-21)

Proof of Proposition 1. At the point of an acquisition $V = V^A_G$, the Nash formula yields

$$p^A(V^A_G) = aV^A_G + \rho(V^A_G - F - d^A(V^A_G) - d^S(V^A_G)),$$

(A-22)

Now we need to determine the value $d^S(V^A_G)$ and $d^A(V^A_G)$ explicitly. First:

$$d^S(V^A_G) = aV^A_G$$

(A-23)

by the assumption that if the asset stays with the seller, it is worth $aV^A_G$. The value of $d^A(V^A_G)$ is

$$d^A(V^A_G) = v^O(A)$$

(A-24)
where \( v^{O*}(V_A^G) \) is defined in (8). In order to find \( V_A^G \), we need to take two cases: \( V_A^G \leq V_O^G \), and \( V_A^G > V_O^G \).

If \( V_A^G \) is lower than \( V_O^G \), which means that it is optimal to start the acquisition before it is optimal to start the internal growth, then at the point of acquisition negotiation, the value of the internal growth, \( v^{O*}(V_A^G) \), is still at the maximum; i.e.,
\[
v^{O*}(V_A^G) = \theta(V_A^G - k^O)(\frac{V_A^G}{V_O^G})^\gamma + (1 - \theta)(\beta V_O^G - k^O)(\frac{V_A^G}{V_O^G})^\gamma ,
\]
and the buyer can walk away from the negotiation, and is still able to exercise the internal growth option optimally later.

However, if the optimal \( V_A^G \) is higher than \( V_O^G \), which means that the acquisition occurs after it is optimal to start the internal growth, then at the point of the negotiation, the internal growth has already started to lose its value. If the negotiation breaks down, it is optimal for the buyer to start the internal growth right away. Hence
\[
v^{O*}(V_A^G) = \theta(V_A^G - k^O) + (1 - \theta)(\beta V_A^G - k^O)(\frac{1}{\beta})^\gamma . \tag{A-25}
\]

As a result, \( p^A(V_A^G) \) is also separated into two cases: \( V_A^G > V_O^G \) and \( V_A^G \leq V_O^G \). To find the optimal \( V_A^G \), in each case we plug \( p^A(V_A^G) \), into (2), and obtain
\[
p^A(V_A^G) = p^A = aV_A^G + \rho(V_A^G - F - v^{O*}(V_A^G) - aV_A^G) . \tag{A-26}
\]

Then differentiate the above equation with respect to \( V_A^G \). Solving the first-order condition yields the results as follows:

For \( V_A^G < V_O^G \), we have
\[
V_A^G = \frac{\gamma}{\gamma - 1} \frac{F}{1 - a} . \tag{A-27}
\]

For \( V_A^G \geq V_O^G \), then:
\[
V_A^G = \frac{\gamma}{\gamma - 1} \frac{\rho(\theta + \beta^{-\gamma}(1 - \theta)k^O + (1 - \rho)F}{\rho(\theta + \beta^{-\gamma}(1 - \theta)) + (1 - \rho)(1 - a)} . \tag{A-28}
\]

It can be verified that the second derivative of equation (2) with respect to \( V_A^G \) is negative, guaranteeing that \( v^A \) obtains its maximum value at \( V_A^G \).

Finally, by comparing \( V_A^G < V_O^G \), and \( V_A^G \geq V_O^G \), we conclude that \( V_A^G < V_O^G \), if and only if
\[
\frac{F}{1 - a} < \frac{\theta + \beta^{-\gamma}(1 - \theta)k^O}{\theta + \beta^{-\gamma}(1 - \theta)} , \tag{A-29}
\]
and \( V_A^G \geq V_O^G \), if and only if
\[
\frac{F}{1 - a} \geq \frac{\theta + \beta^{-\gamma}(1 - \theta)k^O}{\theta + \beta^{-\gamma}(1 - \theta)} , \tag{A-30}
\]
so the two ratios, \( \frac{F}{1 - a} \) and \( \frac{\theta + \beta^{-\gamma}(1 - \theta)k^O}{\theta + \beta^{-\gamma}(1 - \theta)} \), separate the two scenarios.
Proof that $\dot{V}_G^{A*}/\dot{p} < 0$ for the Late Acquisition Case. For $\frac{F}{1-a} \geq \frac{0+\beta^{-\gamma}(1-\theta)}{0+\beta^{-\gamma}(1-\theta)} k^O$, $V_G^{A*} = \gamma \frac{\theta + \beta^{1-\gamma}(1-\theta)k^O - (F(\theta + \beta^{1-\gamma}(1-\theta)))}{\gamma - 1} \frac{\theta + \beta^{1-\gamma}(1-\theta)k^O - (F(\theta + \beta^{1-\gamma}(1-\theta)))}{\gamma - 1}$, differentiate $V_G^{A*}$ with respect to $\dot{\rho}$:

$$\frac{\partial V_G^{A*}}{\partial \rho} = \frac{\gamma}{\gamma - 1} \frac{(1-a)(\theta + \beta^{-\gamma}(1-\theta))k^O - F(\theta + \beta^{1-\gamma}(1-\theta))}{((1-\theta)\beta \rho + \beta^{\gamma}(1-\theta)(1-\theta) - \theta \rho)^2}.$$  
(A.29)

It holds that $\frac{\partial V_G^{A*}}{\partial \rho} \leq 0$ when $(1-a)(1 + (\beta^{-\gamma}(1-\theta)k^O - F(\theta + (\beta^{-\gamma}(1-\theta)k^O)) \leq 0$, which is true for $\frac{F}{1-a} \geq \frac{0+\beta^{-\gamma}(1-\theta)}{0+\beta^{-\gamma}(1-\theta)} k^O$. Hence, $V_G^{A*}$ declines with $\rho$.

Proof of Proposition 2. The buyer and the seller will agree to negotiate only when the surplus is non-negative, and from the above proof, the surplus is lowest when $\rho = 1$. So we identify the minimum level of synergies required for an acquisition when $\rho = 1$.

First recall that when $\rho = 1$, $V_G^{A*} = V_G^{O*}$ and from (5), the surplus is positive only when $V_G^{O*} - F - d_A(V_G^{O*}) - d_s(V_G^{O*}) \geq 0$. Next, substitute $aV_G^{O*}$ for $d_A(V_G^{O*})$ and $\theta(V_G^{O*} - k^O) + (1-\theta)(\beta V_G^{O*} - k^O) (1/\beta)$ for $d_s(V_G^{O*})$, then solve for $\overline{A} \equiv 1 - a$.

To see that $\overline{A}$ increases with $F$, differentiate $\overline{A}$ with respect to $F$:

$$\frac{\partial \overline{A}}{\partial F} = \frac{(\gamma - 1)\gamma k^O C D}{(F(\rho - 1) - (\gamma + \rho - 1)k^O C)^2} \geq 0.$$  
(A.30)

where $C = \theta + \beta^{-\gamma}(1-\theta) > 0$, and $D = \theta + \beta^{1-\gamma}(1-\theta) > 0$.

To see that, for a sufficiently low $F$, $\overline{A}$ declines with $\beta$, it suffices to show that $\frac{\partial \overline{A}}{\partial \beta} \leq 0$. First, differentiate $\overline{A}$ with respect to $\beta$:

$$\frac{\partial \overline{A}}{\partial \beta} = \frac{(C + \gamma F(\gamma - 1) - D F \gamma + C^2 \beta k^O)\beta - (\gamma - 1) \gamma k^O (1-\theta)}{C^2 \gamma k^O}.$$  
(A.31)

The sign of $\frac{\partial \overline{A}}{\partial \beta}$ depends on that of the numerator, which is linear in $F$, and is negative when $F \leq \frac{C^2 \beta k^O}{D \gamma + C \beta (\gamma - 1)}$. So if $F$ is not too large, $\frac{\partial \overline{A}}{\partial \beta} \leq 0$.

Finally, to see that for a sufficiently low $F$, $\overline{A}$ increases with $\sigma$, differentiate $\overline{A}$ with respect to $\sigma$:

$$\frac{\partial \overline{A}}{\partial \sigma} = \frac{\partial \overline{A}(\gamma(\sigma))}{\partial \gamma(\sigma)} \frac{\partial \gamma(\sigma)}{\partial \sigma}.$$  
(A.32)
It can be verified that $\frac{\partial \gamma(\sigma)}{\partial \sigma} \leq 0$. So it suffices to show that $\frac{\partial A(\gamma)}{\partial \gamma} \leq 0$.

$$\frac{\partial A(\gamma)}{\partial \gamma} = -\beta^{-\gamma} k^O (\beta^\gamma C D(-F + C k^O) - \gamma((\beta C - D) F(\gamma - 1) + \beta C^2 k^O)(t - 1) \log(\beta)) \frac{C^2 \gamma k^O}{C^2 k^O}$$  \hspace{1cm} (A-33)

The sign of $\frac{\partial A(\gamma)}{\partial \gamma}$ depends on that of the numerator, which is linear in $F$, and is negative when $F \leq C^2 k^O (\beta^\gamma (\beta C - D) F(\gamma - 1) \log(\beta))$. So if $F$ is not too large, $\frac{\partial A(\gamma)}{\partial \gamma} \leq 0$.

**Proof of Proposition 3.** First, consider the early acquisition case. Equation (12) immediately reveals that $V^A_G$ does not depend on $\beta$. To see that $v^A(V)$ declines with $\beta$, substitute (11) and (12) into (2) and differentiate $v^A(V)$ with respect to $\beta$:

$$\frac{\partial v^A(V)}{\partial \beta} = (\theta - 1) \left( \frac{\beta - 1}{\beta} \right) \left( \frac{\beta(1 + (\beta^{-1} - 1)\theta)}{\gamma k^O} \right)^{\gamma - 1} \times \left( \frac{(\gamma - 1) V}{\beta(1 + (\beta^{-1} - 1)\theta)} \right)^\gamma \leq 0. \hspace{1cm} (A-34)$$

Next, consider the late acquisition case. To see that there is a unique $\beta^*$ such that for $\beta > \beta^*$, $V^A_G$ increases with $\beta$, and for $\beta < \beta^*$ declines with $\beta$, differentiate $V^A_G$ with respect to $\beta$:

$$\frac{\partial V^A_G}{\partial \beta} = \frac{((1 - \theta)\rho \beta^\gamma y(\beta))}{((1 - \theta)\rho \beta + \beta^\gamma(1 - a - (1 - a - \theta)\rho))^2}, \hspace{1cm} (A-35)$$

where,

$$y(\beta) = \frac{k^O(\theta - 1)\rho}{\beta^\gamma} - F(1 - \gamma)(1 - \rho) + k^O \rho(1 - \gamma) + k^O \frac{(1 - a)\gamma - (1 - a - \theta + \beta\theta)\rho}{\beta}$$  \hspace{1cm} (A-36)

Since $0 \leq \rho \leq 1$, $\frac{\partial V^A_G}{\partial \beta}$ can be positive or negative, depending on $y(\beta)$. Differentiating $y(\beta)$ with respect to $\beta$ gives

$$\frac{\partial y(\beta)}{\partial \beta} = k^O \beta^{-2 - \gamma}(\beta \rho(1 - \theta) + \beta^\gamma((1 - a)(1 - \rho) + \theta\rho)) \geq 0. \hspace{1cm} (A-37)$$
Evaluating $y(\beta)$ at $\beta = 1$ gives

$$y(\beta)|_{\beta=1} = k^O((1 - a)\gamma(\rho - 1) - \rho) - F(1 - \gamma)(1 - \rho)$$
$$\leq k^O((1 - a)\gamma(\rho - 1) - k^O(1 - a)(1 - \gamma)(1 - \rho))$$
$$= k^O((1 - \rho)a - 1) < 0,$$  \hspace{1cm} (A-38)

where the inequality follows because for $\beta = 1$, $F \geq (1 - a)k^O$. Next, when $\beta \to \infty$, we have

$$y(\beta)|_{\beta=\infty} = F(\gamma - 1)(1 - \rho) + k^O\theta\rho(\gamma - 1) \geq 0.$$  \hspace{1cm} (A-39)

From (A-33)-(A-35), we conclude there is a unique $\beta^*$ such that $y(\beta) = 0$, and $\frac{\partial v_{A^*}^{A^*}}{\partial \beta} = 0$. For any $\beta < \beta^*$, we have $y(\beta) < 0$, and $\frac{\partial v_{A^*}^{A^*}}{\partial \beta} < 0$. For any $\beta > \beta^*$, we have $y(\beta) > 0$, and $\frac{\partial v_{A^*}^{A^*}}{\partial \beta} > 0$.

To see that $v_{A^*}^{A^*}(V)$ declines with $\beta$, substitute (11) and (12) into (2) and differentiate $v_{A^*}^{A^*}(V)$ with respect to $\beta$:

$$\frac{\partial v_{A^*}^{A^*}(V)}{\partial \beta} = (\theta - 1)\left(\frac{\gamma \rho((\gamma - 1)V)^\gamma}{\beta(\gamma F^B V(1 - \rho) + k^O \gamma(1 + (\beta^\gamma - 1)\theta)\rho)^\gamma}\right)$$
$$\times \frac{F\beta(1 - \rho) - k^O(1 - a - \rho + \theta \rho + \beta \theta \rho)}{(\beta^\gamma(1 + a(\rho - 1) - (1 - \theta)\rho) + \beta(\rho(1 - \theta))^{1-\gamma})}. \hspace{1cm} (A-40)$$

The first term is negative, and the second term is always positive. The sign of $\frac{\partial v_{A^*}^{A^*}(V)}{\partial \beta}$ thus depends on the sign of the third term. Because $\frac{F}{1 - a} \geq \frac{\beta + \beta^{-\gamma}(1 - \theta)}{1 + \beta^{-\gamma}(1 - \theta)} k^O$, it can be verified that

$$\frac{F\beta(1 - \rho) - k^O(1 - a - \rho + \theta \rho + \beta \theta \rho)}{(\beta^\gamma(1 + a(\rho - 1) - (1 - \theta)\rho) + \beta(\rho(1 - \theta))^{1-\gamma})} \geq 0.$$  \hspace{1cm} (A-41)

Hence, $\frac{\partial v_{A^*}^{A^*}(V)}{\partial \beta} \leq 0$.

**Proof of Proposition 4.** To prove that $\hat{v}_{A^*}^{A^*} = \hat{\psi}_{A^*}^{A^*}$, when $\frac{F}{1 - a} < \frac{\beta + \beta^{-\gamma}(1 - \theta)}{1 + \beta^{-\gamma}(1 - \theta)} k^O$, recognize that since $V_{A^*}^{A^*}$ is not a function of $\beta$, $\hat{\psi}_{A^*}^{A^*} = \hat{v}_{A^*}^{A^*} = E[v_{A^*}^{A^*}(V_{A^*}^{A^*})]$, the unconditional expectation of $v_{A^*}^{A^*}(V_{A^*}^{A^*})$, and the announcement effect is zero. When $\frac{F}{1 - a} \geq \frac{\beta + \beta^{-\gamma}(1 - \theta)}{1 + \beta^{-\gamma}(1 - \theta)} k^O$, $V_{A^*}^{A^*} = V_{A^*}^{A^*}(\beta)$. For the proof, we write $v_{A^*}^{A^*}(\beta)$ to explicitly...
show the dependence of $v^{A*}(\cdot)$ on $\beta$, and suppress the dependence of $v^{A*}(\cdot)$ on $V$.

Then:

$$
\hat{v}_t^{A'} = E[v^{A*}(\beta) V^{A*}_G(\beta) > V^m_t]
= E[v^{A*}(\beta) | \beta < G^{-1}(V^m_t)]
= \int_{\beta}^{G^{-1}(V^m_t)} v^{A*}(\beta) \frac{f(\beta)}{F(G^{-1}(V^m_t))} d\beta
= \frac{1}{F(G^{-1}(V^m_t))} \int_{\beta}^{G^{-1}(V^m_t)} v^{A*}(\beta) f(\beta) d\beta
> \frac{1}{F(G^{-1}(V^m_t))} \int_{\beta}^{G^{-1}(V^m_t)} v^{A*}(G^{-1}(V^m_t)) f(\beta) d\beta
= \frac{1}{F(G^{-1}(V^m_t))} v^{A*}(G^{-1}(V^m_t)) \int_{\beta}^{G^{-1}(V^m_t)} f(\beta) d\beta
= v^{A*}(G^{-1}(V^m_t))
= \hat{v}_t^{A}
= \hat{v}_t^{A}(\beta)
= \hat{v}_t^{A}(X)
$$

(A-42)

The inequality in the fifth line follows because $v^{A*}(\cdot)$ declines monotonically with $\beta$ so $v^{A*}(\beta) > v^{A*}(G^{-1}(V^m_t))$ for all $\beta < G^{-1}(V^m_t)$. Also, we use the fact that $V^m_t = V^{A*}_G(\beta)$ and $G^{-1}(V^{A*}_G(\beta)) = \hat{v}_t^{A}$ in the equality in the sixth line.

**Proof of Proposition 5.** To show that $\hat{v}_G^{A} \geq \hat{v}_t^{A}$, we first show that $E[v^{A*}(\cdot)|\beta < X]$ declines with $X$. Again, we write $v^{A*}(\beta)$ to explicitly show the dependence of $v^{A*}(\cdot)$ on $\beta$. Next, differentiate $E[v^{A*}(\beta)|\beta < X]$ with respect to $X$:

$$
\frac{\partial}{\partial X} E[v^{A*}(\beta)|\beta < X] = \frac{\partial}{\partial X} \int_{\beta}^{X} v^{A*}(\beta) \frac{f(\beta)}{F(X)} d\beta
= - \int_{\beta}^{X} v^{A*}(\beta) \frac{f(\beta) f(X)}{F(X)^2} d\beta + v^{A*}(X) \frac{f(X)}{F(X)}
< - \int_{\beta}^{X} v^{A*}(\beta) \frac{f(\beta) f(X)}{F(X)^2} d\beta + v^{A*}(X) \frac{f(X)}{F(X)}
= -v^{A*}(X) \frac{f(X)}{F(X)} + v^{A*}(X) \frac{f(X)}{F(X)}
= 0.
$$

(A-43)
We use Leibniz’s rule in the equality in the second line and the fact that \( v^{A'}(\beta) > v^{A}(X) \) for all \( \beta < X \) in the inequality in third line. Next, write

\[
\hat{v}^A_{i_2} = E[v^{A'}(\beta)|V^A_G(\beta) > V^{mt_2}]
\]

\[
= E[v^{A'}(\beta)|\beta < G^{-1}(V^{mt_2})]
\]

\[
\geq E[v^{A'}(\beta)|\beta < G^{-1}(V^{mt_1})]
\]

\[
= E[v^{A'}(\beta)|V^A_G(\beta) > V^{mt_1}]
\]

\[
= \hat{v}^A_{i_1}.
\] (A-44)

The inequality follows because when \( V^{mt_2} \geq V^{mt_1}, G^{-1}(V^{mt_2}) \leq G^{-1}(V^{mt_1}) \) and \( E[v^{A'}(\beta)|\beta < X] \) declines with \( X \). When \( V^{mt_2} > V^{mt_1} \), and \( G^{-1}(V^{mt_2}) < G^{-1}(V^{mt_1}) \), the inequality is strict.

**Proof of Lemma 2.** Let \( V^*_G \) denote the threshold that maximizes the seller’s utility. The solutions for \( V^*_G \) are divided into two regimes, depending on the relative values of \( \frac{F}{1-a} \) and \( \frac{0+\beta^{-}(1-\theta)}{\theta+\beta^{-}(1-\theta)}k^O \):

1) For \( \frac{F}{1-a} < \frac{0+\beta^{-}(1-\theta)}{\theta+\beta^{-}(1-\theta)}k^O \), substituting \( d^A(V^*_G) \) for \( V < V^*_G \) into (6), and \( V^*_G \)

for \( V^*_G \), and solving the first-order condition for \( V^*_G \) yields

\[
V^*_G = \frac{\gamma F}{\gamma - 1 - a}.
\] (A-45)

It can be verified that the second derivative is negative, so \( V^*_G \) maximizes \( s(V) \). Next, we will verify that the seller has no incentive to wait beyond \( V^*_G \), i.e., it will not choose \( V^*_G > V^{O*}_G = V^{A*}_G \). Substitute \( d^A(V^*_G) \) for \( V \geq V^*_G \) into (6) and solve the first-order condition for \( V^*_G \):

\[
V^*_G = \frac{\gamma F - (\theta + \beta^{-}(1-\theta))k^O}{\gamma - 1 - a - (\theta + \beta^{-}(1-\theta))k^O}.
\] (A-46)

The second derivative of \( s(V) \) with respect to \( V^*_G \) is \( \beta'[\beta^{(1-\theta)} - (\theta + \beta^{-}(1-\theta))k^O] \). Because \( \frac{F}{1-a} < \frac{0+\beta^{-}(1-\theta)}{\theta+\beta^{-}(1-\theta)}k^O \) and \( a \in [0, 1] \), it follows that \( F < (\theta + \beta^{-}(1-\theta))k^O \). This implies that the second derivative is positive, and \( V^*_G \) minimizes \( s(V) \). Therefore, the seller will not choose \( V^*_G > V^{O*}_G \).

2) For \( \frac{F}{1-a} > \frac{0+\beta^{-}(1-\theta)}{\theta+\beta^{-}(1-\theta)}k^O \), substituting \( d^A(V^*_G) \) for \( V = V^*_G \) into (6), and \( V^*_G \)

for \( V^*_G \), and solving the first-order condition for \( V^*_G \) yields the same result as shown in (A-46). However, in this case the second derivative of (6) is negative when \( F > (\theta + \beta^{-}(1-\theta))k^O \). Thus, \( V^*_G \) maximizes \( s(V) \).
Next, we show that \( V_{S}^{G} \geq \gamma \frac{F}{\gamma - 1 (1 - a)} \), the surplus maximizing acquisition threshold. In the first regime, the result is immediate since \( V_{S}^{G} = V_{A}^{G} = \gamma \frac{F}{\gamma - 1 (1 - a)} \). In the second regime, the inequality is true if and only if \( F \geq \frac{\theta + \beta - \gamma (1 - \theta)}{\theta + \beta - \gamma (1 - \theta)} kO \), the condition of the present regime. Because \( V_{S}^{G} \) maximizes the seller’s utility, and \( V_{S}^{G} \geq \gamma \frac{F}{\gamma - 1 (1 - a)} \), the seller’s utility increases strictly monotonically in the acquisition threshold until at least the surplus maximizing acquisition threshold.

**Proof of Proposition 6.** The relative values of \( \frac{F}{1 - a} \) and \( \frac{\theta + \beta - \gamma (1 - \theta)}{\theta + \beta - \gamma (1 - \theta)} kO \) determine the acquisition strategies of the buyer and the seller, so they also determine whether the seller or the buyer starts the negotiation:

1) For \( \frac{F}{1 - a} \leq \frac{\theta + \beta - \gamma (1 - \theta)}{\theta + \beta - \gamma (1 - \theta)} kO \), the buyer’s and the seller’s threshold are the same. So the negotiation can be initiated by either party.

2) For \( \frac{F}{1 - a} > \frac{\theta + \beta - \gamma (1 - \theta)}{\theta + \beta - \gamma (1 - \theta)} kO \), by Lemma 2, the seller has no incentive to start negotiations at least until the socially efficient threshold \( \gamma \frac{F}{\gamma - 1 (1 - a)} \) has passed because its utility increases at least until that point. As we argue in Section 2.7, the buyer’s optimal acquisition threshold, \( V_{B}^{A} \), is lower than the socially efficient threshold, so the buyer will initiate the transaction.

**References**


