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Arbitraging Arbitrageurs

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ABSTRACT

This paper develops a theory of strategic trading in markets with large arbitrageurs. If arbitrageurs are not well capitalized, capital constraints make their trades predictable. Other market participants can exploit this by trading against them. Competitors may find it optimal to lend to arbitrageurs that are financially fragile; additional capital makes the arbitrageurs more viable, and lenders can reap profits from trading against them for a longer time. The strategic behavior of these market participants has implications for the functioning of financial markets. Strategic trading may produce significant price distortions, increase price manipulation, and trigger forced liquidations of large traders.

IN MANY FINANCIAL AND COMMODITY MARKETS a few large players account for a significant fraction of the trading volume. These large players are often arbitrageurs because their primary activity involves taking large positions to profit from small mispricings.

The presence of large players is usually well known to market participants. In some cases this is the result of reputation built over time. Examples include Yasuo Hamanaka, the Sumitomo trader once known as "Mr. Five Percent" among copper traders to reflect its holdings of 5% of the total copper market; Long-Term Capital Management (LTCM), which was referred to as the "Central Bank of Volatility" because of its large short positions in equity derivatives; Metallgesellschaft, which was known as the largest "Wall Street Refiner" after it built positions in oil futures contracts equivalent to the annual production of Kuwait; and more recently, Enron, which was nicknamed "Gas Bank" for its overwhelming trading and risk management activities in electricity and gas markets. In other cases, the holding of large positions is known because of reporting by exchanges through which the positions are held. Derivatives exchanges, for example, regularly publish information on large positions held in the contracts traded on the exchange. An examination of a cross-section of reports published by the London Metal Exchange shows a concentration of open positions in many contracts held by a single market participant with a net position exceeding 50% of the open interest. Although position reports do

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not identify holders, identities are often known to other market participants. Finally, institutions that interact with large players, such as counterparties, clearing brokers, and lenders, know of the presence and identities of the large traders.

In normal circumstances, well-capitalized arbitrageurs act as suppliers of liquidity, and their presence is key to the smooth functioning of markets. When arbitrageurs become financially constrained, however, they cannot take full advantage of mispricings and may even become partly paralyzed; a constraint limits their ability to provide liquidity and also makes it more difficult to control exposure by rebalancing their portfolios. Severely constrained arbitrageurs may have an incentive to move prices in an attempt to ensure their own survival, and in the event of liquidation, they are forced to dump large positions on the market. In this case, arbitrageurs' actions can temporarily cause prices to diverge considerably from values, and the presence of arbitrageurs destabilizes markets.

Sophisticated traders can use knowledge of the financial condition of an arbitrageur to benefit from the predictable price deviations caused by a constrained arbitrageur's trades. When the arbitrageur's financial condition is very weak, other traders provide liquidity and are beneficial to the arbitrageur, because they increase the amount the arbitrageur generates from liquidating its positions. For a healthier but still financially fragile arbitrageur, the trades of sophisticated traders can be detrimental enough to tip the balance against recovery of the arbitrageur, forcing it into insolvency. Actions of these traders push markets against the arbitrageur.

We model an arbitrageur that trades a single asset. The type of arbitrage we have in mind is statistical arbitrage or model-based convergence trading. The arbitrageur interacts with other traders, whom we refer to as strategic traders, that attempt to gain from knowledge of the arbitrageur's financial condition. When the arbitrageur is well capitalized, strategic traders find it optimal to remain inactive. But, as the arbitrageur's financial condition weakens, strategic traders decide to trade. The optimal actions of the arbitrageur and of the strategic traders depend on the characteristics of the market in which the asset is traded, and on the arbitrageur's financial condition and the size of its positions.

A binding capital constraint affects the arbitrageur's trades such that it is one of the following types: (1) partial reduction of the arbitrageur's position, (2) manipulative increase in the arbitrageur's position, or (3) full liquidation. If an arbitrageur is in danger of violating minimum capital requirements, liquidation of some outstanding positions is the most obvious way to regain control. In less than perfectly liquid markets, this is costly, as liquidation trades have an adverse impact on price and, through price, on the arbitrageur's capital. The price impact creates an incentive for strategic traders to take the other side of the trade and by absorbing part of the trade helps improve the arbitrageur's financial condition.

In thin markets, or for arbitrageurs with large positions, the effect of the adverse price impact can be so great that partial liquidation of positions is no longer viable. Then, the arbitrageur must either fully liquidate its positions or move the asset price in an attempt to remain solvent. Price manipulation is costly. It involves trading at an unfavorable price—an arbitrageur with a long position needs to buy more of the asset to keep the price high, whereas an arbitrageur with a short position needs to sell more to keep the price low. Again, deviations of price from value create an incentive for strategic traders to enter the market and trade against the arbitrageur. In this case, strategic traders raise the cost to the arbitrageur of manipulating the price. The arbitrageur chooses to fully liquidate positions when the cost of manipulating the price exceeds the cost associated with full liquidation.

A strategic trader bases trades on its expectation of the arbitrageur's actions. In thin markets, when there is potential for a considerable error in forecasting the arbitrageur's choice of action, because it may either support the price to stay afloat or liquidate, the strategic trader prefers to stay inactive. A particular choice of action results in a profit for the strategic trader if the arbitrageur chooses to trade one way but in a loss if the arbitrageur decides to trade the other way. In this case, the strategic trader may find it optimal to lend to the arbitrageur, even when the loan per se has a negative expected return. A loan, by reducing the error in predicting the arbitrageur's actions, increases the strategic trader's trading profit. The strategic trader gains from selling the asset at the artificially high prices that result as an arbitrageur with a long position buys more of the asset in order to survive. In addition, the strategic trader benefits from the lower prices that result as the arbitrageur trims positions back to more normal levels.

Our work relates to research that recognizes the importance of financial constraints for trading in financial markets. Shleifer and Vishny (1997) argue that, in general, potential investors of trading institutions observe only returns but not the investment opportunities available. Traders are then not always able to raise additional capital, and financial constraints become binding. In a liquid market, this leads to the inefficient liquidation of positions. Attari and Mello (2001) and Xiong (2001) take into account the fact that in illiquid markets, individual traders influence asset prices. They show that financial constraints on traders in such markets can be responsible for periods of excessive volatility and even liquidity crises. All these papers ignore the possibility that other traders can take advantage of the restricted financial flexibility of large arbitrageurs, which significantly increases the risks that arbitrageurs face.

The models of predatory behavior in securities markets in Brunnermeier and Pedersen (2004) and Pritsker (2003) are closest to the one presented here. In Brunnermeier and Pedersen (2005), a financially fragile trader is assumed to be forced to liquidate its position. Liquidating the whole position at once is suboptimal for the trader because it faces a convex price impact function. The knowledge of a gradual liquidation allows predatory traders to take advantage of future downward price pressure by trading in the same direction as the fragile trader. In contrast, in this paper the trading opportunity for strategic traders arises because of the combination of capital requirements and limited market liquidity rather than the shape of the price impact function. Given that liquidation is endogenous in our model, we are able to assess the effect of strategic trading on the probability of liquidation by arbitrageurs. Also, our paper places explicit emphasis on the funding of traders whereas Brunnermeier and Pedersen's focus is on price dynamics and its consequences. As do Brunnermeier and Pedersen (2005), Pritsker (2003) studies the dynamics of prices in a setting in which some agents have to liquidate positions. He assumes a setting with heterogeneous risk-averse traders, whereas both Brunnermeier and Pedersen as well as this paper assume risk neutrality of traders. Cai (2002) documents strategic trading by participants in T-bond futures markets during the collapse of LTCM in the second half of 1998. She finds that market participants, by selling ahead of LTCM, drove prices down and benefited from closing out the resulting short positions at lower prices. Consistent with her results, our model predicts that large collapses begin with liquidations by strategic traders.

Taking advantage of the financial fragility of competitors has been analyzed in papers that link product markets and corporate finance. Bolton and Scharfstein (1990) show that a firm may engage in predatory price reductions if its competitor is financially constrained. This occurs when an increase in future profits more than compensates for temporarily low current profits. We also link the plan of forcing a collapse and the importance of financing constraints to the success of such a trading strategy. In our case, the motivations and the actions of participants are different. In Bolton and Scharfstein (1990), the predator benefits from increased market power after the collapse of the rival. In our approach, the predator takes advantage of depressed prices *during* the time of the collapse, as well as the arbitrageur's price manipulation efforts *before* the collapse. We also see different behaviors of the aggressors. Strategic traders may decide to lend to a financially fragile arbitrageur so it can continue trading for a longer period. We relate features typically seen in corporate finance to trading environments and thus take account of aspects often overlooked in explaining the behavior of traders.

Section I describes the model used in Sections II.A and II.B to analyze market activity in two polar cases, namely, when the arbitrageur has complete financial flexibility, and when it does not have any financial flexibility. In Section II.C the results are derived for an arbitrageur whose flexibility is between the two polar cases. In Section III we analyze strategic trader lending to the arbitrageur to improve its trading profit. A number of extensions are presented in Section IV. Section V concludes.

I. The Model

Consider a market for an asset that is less than perfectly liquid. The market price of the asset in this case is affected by the trading decisions of market participants. At the initial date, $t = 0$, traders are assumed to have positions in the asset that are accumulated from previous actions. We analyze trading decisions at dates $t = 1$ and $t = 2$.

The fundamental value of the asset is assumed to be constant over time and is denoted by I. More complex value processes can be accommodated without affecting the main results. The market price of the asset at time t is denoted by P_t. In each period, the price of the asset is set by a market-clearing condition and may deviate from its value.

There are three groups of traders in the market: liquidity traders, a single arbitrageur, and a single strategic trader.

The aggregate demand of a large number of small liquidity traders is given by $D_t^L = \varepsilon_t - \beta(P_t - I)$, where D_t^L is the flow of liquidity trades in the interval $(t-1, t)$, and $\beta > 0$. Liquidity demand is composed of two parts. A random component, ε_t, which is independent and identically uniformly distributed on the interval $[-R, R]$, represents pure noise trading activity. The second term, $-\beta(P_t - I)$, takes into account that liquidity traders respond to deviations of the market price of the asset from its value. As prices fall, more buyers are drawn to the market; as prices rise, more sellers appear. We refer to this group of liquidity traders as value traders, since they sell the asset when its price is higher than its intrinsic value and purchase it when the price is lower. The coefficient $\beta < \infty$ is an indicator of market depth; the larger β is, the greater the sensitivity of value trader demand to price deviations.

In the absence of additional market participants, market clearing implies $P_t = I + \frac{\varepsilon_t}{\beta}$. Thus, the price at time t, P_t, is independent and identically distributed with a mean of I. This assumes that the impact of the liquidity shock dies out completely in one period and has no impact on future prices, which is a reasonable specification for pure liquidity shocks. It is also possible to model liquidity shocks and price deviations as following persistent or mean-reverting processes. Our results are based on the predictable deviations of prices caused by the trades of a constrained arbitrageur. The presence of other sources of predictability in price deviations does not qualitatively affect our results. All proofs are in the Appendix.

A. The Arbitrageur

A risk-neutral arbitrageur will trade the asset with the objective of maximizing its expected wealth at $t = 2$. Let θ_{t-1} be the arbitrageur's position in the asset after trading at time $t-1; \theta_t - \theta_{t-1}$ is the quantity that the arbitrageur trades at time t. In addition to holding θ_t units of the asset, the arbitrageur has debt outstanding of B_t. The initial condition of the arbitrageur is characterized by θ_0 and B_0. For expositional clarity, we assume that the arbitrageur initially holds a long position, that is, $\theta_0 > 0$ and $B_0 > 0$. Restricting our attention to positive values of θ_0 is not critical since the model setup is symmetric.

In each period the arbitrageur observes noise trader activity, ε_t. The transitory nature of the liquidity shock provides the arbitrageur with a valuable timing option. The arbitrageur uses its knowledge of the behavior of value traders and the trading strategies of the strategic trader to determine the optimal quantity to trade to maximize its expected wealth at $t = 2$. We work with a simple version of the problem, assuming that the arbitrageur liquidates its holdings over two periods, $\theta_2 = 0$, and chooses the position at $t = 1, \theta_1$, to maximize its wealth.[1] This eliminates the problem of determining the holdings at time 2 and gives us an intuitive framework to analyze the factors driving the results. The results remain unchanged if we allow for a general value of θ_2, which can, for example, be derived as an optimal value by an arbitrageur with a long horizon. We discuss this in Section IV.

The arbitrageur then decides on its optimal liquidation policy,

$$\max_{\theta_1} E_0[-B_0 - (\theta_1 - \theta_0)P_1 + \theta_1 P_2], \tag{1}$$

which represents the arbitrageur minimizing the cost of liquidating its position over the two periods.

We assume unlimited liability on the part of the arbitrageur to ensure that our results are not driven by the risk-taking incentives that limited liability would induce.

In accordance with common practice in capital markets, the arbitrageur faces a capital constraint that requires it to fund positions partially with its own capital. Even if a collapsed arbitrageur is ultimately able to repay the loans in full, creditors may face significant costs in recovering the loans. This cost comes, for example, in the form of legal fees, time spent in negotiations with the arbitrageur and other creditors, as well as the opportunity cost of forgone returns on the loaned funds.[2] For example, after its forced liquidation, the high-yield debt security house of Drexel Burnham Lambert was able to repay its creditors and counterparties completely but only after lengthy and very costly court proceedings. Therefore, creditors typically require that a certain percentage of the initial investment be financed with equity.[3]

Capital constraints take the form of either position-based constraints, as, for example, requirements on derivatives contracts, or price-based constraints, as, for example, constraints on bank loans. In this paper, we assume that the arbitrageur must satisfy the constraint

$$\theta_t P_t - B_t \geq |\theta_t| M, \tag{2}$$

where $M > 0$ denotes the amount of own capital required per unit position. The left-hand side of equation (2) is the net value of the arbitrageur's position at market prices, whereas the right-hand side is the total capital required. The total capital required is proportional to the size of the position held, θ_t, but is also related to the value of the asset, given that M can be understood as a fraction of the intrinsic value of the asset, $M = mI$. Alternatively, M can be proportional to an average of historical prices of the asset or, simpler, the initial price, P_0. In

[1] Often hedge funds and other arbitrageurs decide to take positions for ex ante specified periods of time.

[2] Imposing a capital requirement has other advantages for a lender. For example, a capital requirement can indicate the quality of a trader, since it forces the trader to raise equity capital from other investors.

[3] When no or little initial investment is needed to enter a position, for example, in the case of forward or futures, counterparties and exchanges demand the deployment of assets as collateral. This is structurally equivalent to a capital requirement imposed by a lender.

any case, M reflects the creditors' demand of an equity capital requirement.[4] In Section IV.A we discuss alternative capital constraints.

The arbitrageur's debt, B_t, changes from one period to the next as the arbitrageur's position in the asset changes. Without loss of generality we assume a zero interest rate. If there are no intermediate additions or withdrawals of wealth, we can write the evolution of the amount owed by the arbitrageur as

$$B_t = B_{t-1} + (\theta_t - \theta_{t-1})P_t. \tag{3}$$

Assume that the strategic trader trades x_t units at t. The price of the asset, set by the market-clearing condition $\theta_t - \theta_{t-1} + x_t + D_t^L = 0$, is

$$P_t = I + \frac{\theta_t - \theta_{t-1}}{\beta} + \frac{x_t + \varepsilon_t}{\beta}. \tag{4}$$

On substituting for P_1 and B_1 using (4) and (3) and rearranging terms, (2) at date 1 can be written as

$$\theta_0\left(I + \frac{\theta_1 - \theta_0}{\beta} + \frac{x_1 + \varepsilon_1}{\beta}\right) - B_0 \geq |\theta_1|M. \tag{5}$$

The arbitrageur's trade to meet the constraint is a function of $\theta_0, \beta, B_0, \varepsilon_1$, and x_1. The left-hand side is the arbitrageur's capital evaluated at the market price of the asset. The term $\frac{\theta_0}{\beta}(\theta_1 - \theta_0)$ on the left-hand side reflects the impact of the arbitrageur's own trades on its own capital. The price impact from reducing positions decreases the equity available to satisfy the capital requirement. The magnitude of the effect increases with the size of the initial position relative to the liquidity of the market, $\frac{\theta_0}{\beta}$. A sale of each unit of the asset depresses the value of the units remaining in the portfolio. This negative effect on the value of the arbitrageur's equity is more severe the larger the remaining portfolio. A large reduction in the equity value per unit liquidated may make it even impossible for the arbitrageur to meet the capital requirement by reducing its position. Concretely, if the initial position is sufficiently high relative to market liquidity, $\theta_0 > \beta M$, an arbitrageur that trades to fulfill the capital requirement cannot do so by liquidating part of its positions.[5] Solving for θ_1 reveals that in this case, the capital requirement imposes a bound below which the arbitrageur's position cannot fall,

[4] This formulation implies that the capital requirement does not fluctuate with short-term deviations of prices from the intrinsic value. This is an assumption made in the literature on trading with capital constraints; see Liu and Longstaff (2004) and Cuoco and Liu (2001). Modeling the capital requirement as a percentage of the new investment made in the asset at each point in time introduces path dependence and complicated nonlinearities that make the analysis intractable. We believe that the main results in the paper would still hold under other forms of capital constraints.

[5] For example, Lowenstein (2000, p. 169) argues that LTCM faced this dilemma. He writes, "The frightful size of its positions put the partners [of LTCM] in a terrible bind. If they sold even a tiny fraction of a big position—say, of swaps—it would send the price plummeting and reduce the value of all the rest."

$$\theta_1 \geq \frac{\theta_0(\theta_0 - x_1 - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)}. \tag{6}$$

When the arbitrageur faces a lower bound it may be forced to liquidate less than desired in order to meet the capital requirement. Depending on the parameters and the decisions by the other market participants, the constraint may even impose that the required θ_1 be larger than the initial position, θ_{t-1}. Then, the arbitrageur has to increase its position in order to fulfill the requirement. Buying has a positive effect on the price, which increases the market valuation of the assets already in the arbitrageur's portfolio.

For initial positions that are small relative to market depth and the per unit capital requirement, $\theta_0 < \beta M$, the constraint places an upper bound on the arbitrageur's position at date 1,

$$\theta_1 \leq \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x_1 - \varepsilon_1)}{(\beta M - \theta_0)}. \tag{7}$$

A negative value for the expression to the right of the inequality sign indicates that the constraint cannot be met, and the arbitrageur is forced to fully liquidate its positions and exit the market, $\theta_1 = 0$. Otherwise, the arbitrageur can satisfy a binding capital constraint by limiting the size of its positions. Because of the price impact of its trades, the arbitrageur has to liquidate more than what it would have to if the market were perfectly liquid.

B. The Strategic Trader

The strategic trader has information on the arbitrageur's financial condition and knowledge of the trading strategy of the arbitrageur and value traders. The strategic trader does not observe the liquidity shock experienced by noise traders. Because θ_1 depends on the liquidity shock, the arbitrageur's strategy is random from the perspective of the strategic trader at $t = 1$. Despite this informational disadvantage, the strategic trader may trade on information regarding the arbitrageur's financial condition. Specifically, the strategic trader knows the arbitrageur's position in the asset as well as its financial condition (borrowings). The extent of the arbitrageur's positions may be known because the arbitrageur is known to hold large positions in the asset, because the size of its trades make it visible to others, or because the exchange publishes information on large positions. The arbitrageur's financial condition can become known in a number of different ways. Observed adverse price movements along with knowledge of the arbitrageur's positions may reveal financial difficulties, information may become available if the arbitrageur attempts to raise additional capital, or information may be revealed by an attempt to reduce positions. For arbitrageurs that are part of publicly held companies, the financial condition may also become known through the release of accounting information.

Strategic traders include banks that trade with and lend to arbitrageurs, institutions that act as clearing brokers for arbitrageurs, and more generally, traders in assets frequently traded by arbitrageurs. These institutions understand the workings of financial markets well represented by their knowledge of trading strategies but may not have a high level of expertise in all assets, which we capture through their lack of knowledge of the liquidity shock. Thus, our analysis includes markets in which few traders play a dominant role.

If it could, the strategic trader would wait to see whether an adverse realization of the liquidity shock for the arbitrageur created a profitable trading opportunity. However, in our setting the strategic trader faces uncertainty with respect to the effect of the liquidity demand. Assuming that strategic traders have less information than arbitrageurs allows us to show that the pool of potential strategic traders is wider than at first thought, consisting of more than just the few that specialize in the same markets or trading strategies as the arbitrageur. According to Cai (2002), a good number of market makers and futures brokers took advantage of LTCM.[6] If we can show that informationally disadvantaged strategic traders can profit from trading with better-informed arbitrageurs, then their profits increase when strategic traders have the same information as the arbitrageur, because they themselves become arbitrageurs.

For a similar reason we consider the case in which the strategic trader is limited to simple trading strategies—strategies such that the strategic trader puts on a position at $t = 1$ and has to liquidate the position at $t = 2$. Again, allowing the strategic trader a richer set of trading strategies will improve the profitability of trading. In this sense we bias the model against finding a role for strategic traders, making our results more robust.

The strategic trader sets up positions that are reversed one period later. If it trades x at $t = 1$, its profits are given as $x(P_2 - P_1)$. Substituting for P_1 and P_2 using (4) allows us to write the strategic trader's objective function as

$$\max_x \frac{x}{\beta}(\theta_0 - 2E[\theta_1] - 2x). \tag{8}$$

The expected profit has a maximum at x^* that solves

$$\theta_0 - 2E[\theta_1] - 4x^* = 0. \tag{9}$$

Given that it does not observe the liquidity demand, the strategic trader is uncertain about the arbitrageur's action. Therefore, the best it can do is to rely on the expectation of the arbitrageur's trading given the distribution of the liquidity shock. Since θ_1 is a function of x^*, it is not possible to solve immediately for the optimal amount of strategic trading.

The strategic trader may also consider lending to the arbitrageur. Lending is common among traders who provide collateral to one another for trading purposes. Later, in Section III, we discuss the incentives for a strategic trader to lend to a financially constrained arbitrageur.

[6] Also, Lowenstein (2000, p. 174) writes that institutions that hadn't been trading before in LTCM's markets entered as they learned about LTCM's financial difficulties: "American International Group, which hadn't shown any interest in equity volatility before, was suddenly bidding for it." Lowenstein goes on to claim that this sudden interest could be aimed only at exploiting LTCM's distress. He also mentions that it was well known during the crisis that Salomon Smith Barney "was, and had been, pounding the fund's positions for months."

II. Optimal Liquidation by the Arbitrageur

As stated, we consider the problem outlined previously where the arbitrageur has to liquidate its initial position over two periods, so that $\theta_2 = 0$. Also, we assume for notational simplicity that the arbitrageur's initial position is large enough to avoid short positions at date 1. As it turns out and is shown here, this requires a level of θ_0 larger than $\frac{1}{2}R$ where R is the upper bound of the liquidity shock produced by the actions of noise traders.

The arbitrageur's problem obtained after substituting for P_1 and P_2 using (4) in (1) is

$$\max_{\theta_1} \theta_0 I - B_0 - (\theta_1 - \theta_0)\left(\frac{\theta_1 - \theta_0}{\beta} + \frac{x + \varepsilon_1}{\beta}\right) - \theta_1\left(\frac{\theta_1}{\beta} + \frac{x}{\beta}\right) \tag{10}$$

$$\text{s.t. } \theta_0\left(I + \frac{\theta_1 - \theta_0}{\beta} + \frac{x_1 + \varepsilon_1}{\beta}\right) - B_0 \geq |\theta_1| M. \tag{11}$$

We start by considering two polar cases, namely, that of an unconstrained arbitrageur and that of an arbitrageur that is forced to liquidate at $t = 1$. We then analyze intermediate cases, wherein the constraint is at times binding and the arbitrageur may be forced to fully liquidate at $t = 1$. In the intermediate cases, we need to separately consider an arbitrageur with a small initial position and with a large initial position. To keep the notation simple, we use lowercase letters for a small initial position and capital letters to indicate a large initial position.

A. Strategic Trading When the Arbitrageur Is Financially Unconstrained

If the arbitrageur is unconstrained, we obtain that the arbitrageur's best response to a given liquidity shock and an order by the strategic trader at $t = 1$ is

$$\theta_1 = \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right). \tag{12}$$

The expected value of θ_1 is $E[\theta_1] = \frac{1}{2}(\theta_0 - x)$. Using (9) we obtain $x^* = 0$. The strategic trader decides not to trade against the arbitrageur. The strategic trader's need to reverse the trade at $t = 2$, combined with the price impact of its trades, makes it costly for it to trade against the arbitrageur. In response to a trade by the strategic trader, an arbitrageur with financial flexibility can adjust the quantities traded at $t = 1$ and $t = 2$ to reduce the cost of liquidating its holdings.

The equilibrium is characterized by $x^* = 0$ and $\theta_1^* = \frac{1}{2}(\theta_0 - \frac{1}{2}\varepsilon_1) > 0$ for $\theta_0 > \frac{1}{2}R$. In the absence of a liquidity shock, the arbitrageur would sell one half of its initial position each period. With positive demand by liquidity traders, $\varepsilon_1 > 0$, the arbitrageur can sell more, since the liquidity demand provides a cushion to the price impact created by the arbitrageur's selling. With negative liquidity demand, $\varepsilon_1 < 0$, the arbitrageur sells less as both its own trade and the liquidity trades push the price in the same direction. Given the restriction on the size of liquidity demand, $\theta_0 > \frac{1}{2}R$, the arbitrageur never buys the asset. The equilibrium price of the asset at date 1 is given by $P_1^* = I - \frac{\theta_0}{2\beta} + \frac{3\varepsilon_1}{4\beta}$.

This result is important since it shows that a strategic trader cannot take advantage of liquidation by a well-capitalized arbitrageur. In fact, the arbitrageur could use trading by the strategic trader to lower the cost of liquidating its positions. This can be seen by substituting (12) into (10), which shows that in expectation the unconstrained arbitrageur's wealth increases with the amount traded by the strategic trader.

B. Strategic Trading When the Arbitrageur Fully Liquidates at $t = 1 (\theta_1 = 0)$

When the arbitrageur's financial situation is very precarious, it always fully liquidates its positions and exits the market at $t = 1$. This happens either because it is forced to do so due to an inability to satisfy the constraint or because it optimally chooses to do so because the cost of meeting the constraint at $t = 1$ exceeds that of liquidation at $t = 1$.

In this case, the equilibrium is characterized by $x^* = \frac{\theta_0}{4}$ and $\theta_1^* = 0$. The equilibrium price of the asset at date 1 is given by $P_1^* = I - \frac{3\theta_0}{4\beta} + \frac{\varepsilon_1}{\beta}$.

The strategic trader has information about the financial condition of the arbitrageur and anticipates the rapid liquidation and the resulting downward price pressure. To take advantage of the temporary drop in the price of the asset, it builds up a long position at date 1. The larger the arbitrageur's position in the asset, the more dramatic the expected mispricing caused by the liquidation, and consequently the greater the strategic trader's demand for the asset.

C. Strategic Trading When the Arbitrageur Is Financially Constrained

In more interesting cases, the arbitrageur is constrained but does not always fully liquidate at $t = 1$. In these cases, the extent of the liquidity shock, ε_1, determines whether the arbitrageur's trade (1) is unaffected by the constraint, (2) is specified to fulfill the constraint, or (3) is a full liquidation.

The constraint has a different effect depending on the size of the position held by the arbitrageur. First, we consider the case in which the arbitrageur has a small position relative to market liquidity, $\theta_0 < \beta M$. Then, we consider the case in which the arbitrageur has a large position, $\theta_0 > \beta M$.

C.1. Arbitrageur with Small Initial Position Relative to Market Liquidity, $\theta_0 < \beta M$

When the arbitrageur has a small initial position, $\theta_0 < \beta M$, the constraint places an upper bound on the position that it can hold after trading at $t = 1$, θ_1. In this case, the arbitrageur can fulfill the capital requirement by liquidating a portion of its holdings. If the constraint is not binding, the arbitrageur trades as an unconstrained arbitrageur would trade, yielding $\theta_1^U = \frac{1}{2}(\theta_0 - x - \frac{1}{2}\varepsilon_1)$. If the constraint is binding, the arbitrageur trades to satisfy the constraint, giving

$\theta_1^C = \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x - \varepsilon_1)}{(\beta M - \theta_0)}$. The values of ε_1 for which the constraint is binding can be obtained by comparing θ_1^C and θ_1^U; the constraint is binding for all ε_1 that satisfy $\bar{l} \equiv 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)} > \varepsilon_1$. Note that for some values of ε_1 it is not possible for the arbitrageur to meet the constraint $\theta_1^C < 0$, and it must trade down to a flat position and exit the market immediately. The values of ε_1 for which this is the case satisfy $\underline{l} \equiv \frac{\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\theta_0} > \varepsilon_1$. This leads to

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if } \varepsilon_1 \geq \bar{l} \\ \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x - \varepsilon_1)}{(\beta M - \theta_0)} & \text{if } \bar{l} > \varepsilon_1 \geq \underline{l} \\ 0 & \text{if } \underline{l} > \varepsilon_1. \end{cases} \tag{13}$$

We solve for the optimal quantity of the strategic trader, x^*, by first obtaining the expected size of the arbitrageur's position after trading at $t = 1, E[\theta_1]$, and using this in (9) to obtain the following result.

PROPOSITION 1: *Suppose that the arbitrageur holds a small initial position relative to market liquidity, $\theta_0 < \beta M$. Then, ordered in increasing levels of the arbitrageur's initial debt, B_0, we have five regions.*

- *(i) In region I, where the probability that the arbitrageur is unconstrained is 1, the strategic trader does not trade, $x_I^* = 0$.*
- *(ii) In region II, where the probability that the constraint is binding and the probability that the arbitrageur is unconstrained are both greater than zero and add up to 1, the strategic trader acquires the amount $x_{II}^* > 0$.*
- *(iii) In region III, where (a) the constraint on the arbitrageur binds with probability 1, the strategic trader acquires the amount $x_{IIIa}^* > 0$; or (b) where the probabilities that the arbitrageur is unconstrained, constrained, or forced to fully liquidate are all greater than zero and add up to 1, the strategic trader acquires the amount $x_{IIIb}^* > 0$.*
- *(iv) In region IV, where the probability that the constraint is binding and the probability that the arbitrageur is forced to fully liquidate are both greater than zero and add up to 1, the strategic trader acquires the amount $x_{IV}^* > 0$.*
- *(v) In region V, where the probability that the arbitrageur is forced to fully liquidate is equal to 1, the strategic trader acquires the amount $x_V^* = \frac{\theta_0}{4} > 0$.*

  The expressions for $x_{II}^*, x_{IIIa}^*, x_{IIIb}^*, x_{IV}^*$, and the cutoff debt levels that separate the regions, $u_I, u_{II}, u_{III}, u_{IV}$, are presented in the Appendix.

The characteristics of region III depend on the support of the distribution of ε_t. If the support is relatively wide, (b) prevails, and (a) otherwise.

Using the proposition, we get additional results.

COROLLARY 1: *Suppose that the arbitrageur holds a small initial position, $\theta_0 < \beta M$. In regions II, III, and IV in Proposition 1, the strategic trader's optimal*

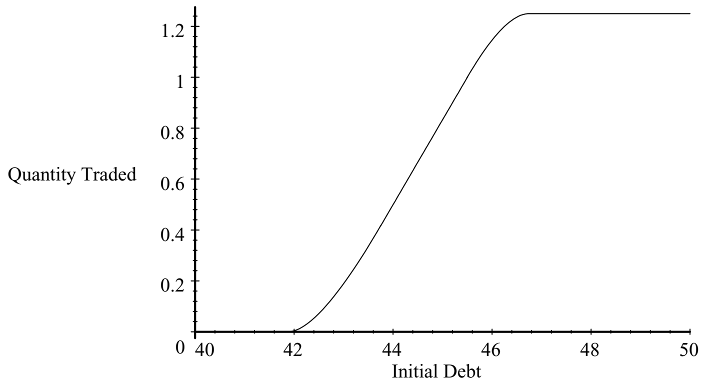

Figure 1. Strategic trader's optimal quantity for varying B_0; arbitrageur holds small position. Parameter values: Initial position of the arbitrageur $\theta_0 = 5$, maximum absolute size of the liquidity shock $R = 0.5$, market depth $\beta = 5$, capital required per unit position $M = 2$, and fundamental value of the asset $I = 10$. This implies for the cutoff points of initial debt $u_I = 41.875, u_{II} = 43.75, u_{III} = 45.5$, and $u_{IV} = 46.75$.

quantity increases with the arbitrageur's debt, $\frac{\partial x^*}{\partial B_0} > 0$, *and remains constant in region V.*

When the strategic trader faces an arbitrageur with a small initial position, its expected profit increases with the arbitrageur's initial borrowing, B_0, and has a maximum value of $\frac{2}{\beta}(\frac{\theta_0}{4})^2$ for levels of B_0 where the arbitrageur is always forced to fully liquidate. For B_0 such that the arbitrageur is always unconstrained, the strategic trader does not trade, and its expected profit is zero. For all B_0 such that the arbitrageur is constrained with probability greater than zero, the strategic trader buys the asset. The arbitrageur always sells the asset. This can be seen by using expression (12) in the case of the unconstrained arbitrageur, given that $\theta_0 > R/2$. A constrained arbitrageur sells more than an unconstrained one to satisfy the upper position bound imposed by the capital constraint.

Figure 1 plots the optimal quantity that the strategic trader trades as a function of the arbitrageur's borrowings. When the arbitrageur's debt is small, the strategic trader is unable to benefit from its information about the arbitrageur's financial situation. This changes as soon as the debt level is high enough to impair the arbitrageur's financial flexibility. The strategic trader then buys the asset in anticipation of a liquidation of the arbitrageur's position to ensure that the constraint is met, and benefits from the resulting price pressure.

COROLLARY 2: *Suppose that the arbitrageur holds a small initial position,* $\theta_0 < \beta M$. *Then, given* θ_0 *and* B_0 *for the arbitrageur, the strategic trader's purchase in* $t = 1$ *(i) reduces the probability of complete forced liquidation of the arbitrageur's holdings at* $t = 1$; *(ii) reduces the amount the arbitrageur is forced to liquidate to meet the constraint; and, (iii) results in higher prices.*

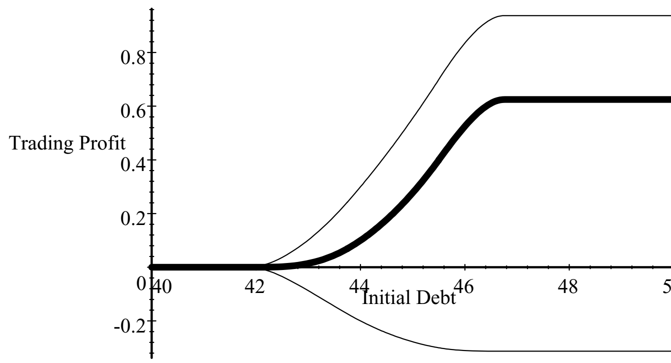

Figure 2. Strategic trader's trading profit for varying B_0; arbitrageur holds small position. The thick line represents the expected total trading profit computed as $xE[(P_2 - P_1)]$; the uppermost line is the expected first-period book profit computed as $x(I - E[P_1])$; and the lowermost line is the expected second-period book profit computed as $-x(I - E[P_2])$. Parameter values: initial position of the arbitrageur $\theta_0 = 5$, maximum absolute size of the liquidity shock $R = 0.5$, market depth $\beta = 5$, capital required per unit position $M = 2$, and fundamental value of the asset $I = 10$. This implies for the cutoff points of initial debt $u_I = 41.875, u_{II} = 43.75, u_{III} = 45.5$, and $u_{IY} = 46.75$.

By buying, the strategic trader provides price support and diminishes the probability that the arbitrageur violates the capital constraint. This reduces the likelihood that the arbitrageur will have to liquidate its entire position. Thus, the strategic trader helps to prevent disruptions in the market by allowing the arbitrageur to liquidate positions over a longer period. An arbitrageur that has a small position benefits from the presence of the strategic trader, since fulfilling the capital constraint requires a lower level of liquidation. The strategic trader only seemingly acts altruistically, since it trades to maximize its own profit.

Figure 2 plots the strategic trader's expected trading profit as a function of the arbitrageur's initial debt level. It documents that the strategic trader's profits stem entirely from gains at $t = 1$, when it takes advantage of the liquidation of positions by the arbitrageur in an attempt to meet the constraint that leads to downward pressure on prices. At $t = 2$, the strategic trader closes out the positions accumulated at $t = 1$ and sells, and this has a downward impact on the price, which represents a cost it pays to trade the illiquid asset.

C.2. Arbitrageur with Large Initial Position Relative to Market Liquidity, $\theta_0 > \beta M$

When the arbitrageur has a large initial position, $\theta_0 > \beta M$, the fall in the value of its holdings caused by the downward pressure on prices resulting from a liquidation can be large enough to make it impossible for the arbitrageur to meet the constraint. This implies that the arbitrageur can fulfill the constraint only if its position remains sufficiently large. If the constraint is not binding, the arbitrageur trades as an unconstrained arbitrageur would, which yields $\theta_1^U = \frac{1}{2}(\theta_0 - x - \frac{1}{2}\varepsilon_1)$. When the constraint is binding, the arbitrageur has two alternatives, to trade to satisfy the constraint, or to liquidate and exit the market. The capital requirement (6) can be solved for θ_1. Meeting it implies

$$\theta_1^C = \frac{\theta_0(\theta_0 - x - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)}. \tag{14}$$

The capital constraint is binding only if the arbitrageur intends to hold a smaller position than it would in the absence of the capital requirement. Comparing θ_1^C to θ_1^U shows that the constraint is binding for $\bar{L} \equiv 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)} > \varepsilon_1$.

An arbitrageur with a large position can always meet the capital requirement by placing a sufficiently large buy order. This will help to keep the price up. However, meeting the constraint this way at $t = 1$ is costly, because the arbitrageur has to liquidate a larger position at $t = 2$. This cost increases with θ_1 and keeps the arbitrageur from pursuing such a strategy without restriction. If the arbitrageur liquidates its entire position at $t = 1$, the net value given in (1) is $\theta_0(I - \frac{\theta_0}{\beta} + \frac{x + \varepsilon_1}{\beta}) - B_0$. Comparing this value with the value for $\theta_1 > 0$ yields a critical θ_1 above which meeting the constraint results in a value lower than full liquidation. Concretely, satisfying the constraint yields an equal or higher value than immediately liquidating when

$$\theta_0 - x - \frac{\varepsilon_1}{2} \geq \theta_1^C. \tag{15}$$

To understand the arbitrageur's behavior, suppose that the strategic trader is not trading and there is no liquidity shock. If $\theta_0 < \theta_1^C$ the arbitrageur would be required to buy the asset to stay solvent, but it does not do so because at $t = 2$ it would have to liquidate a larger amount. However, the arbitrageur is willing to buy if $x + \frac{\varepsilon_1}{2} < 0$ because price pressure from noise and strategic traders makes immediate liquidation more expensive.[7] However, its willingness to buy is limited by the fact that the more the other traders sell, the higher will be the position the arbitrageur needs to hold to fulfill its capital requirement (see equation (14)).

Using (14) for θ_1^C in (15) yields an explicit expression for ε_1 below which full liquidation at $t = 1$ is the optimal choice, $\underline{L} \equiv 2\frac{\beta M(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} > \varepsilon_1$.

This leads to

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if } \varepsilon_1 \geq \bar{L} \\ \frac{\theta_0(\theta_0 - x - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)} & \text{if } \bar{L} > \varepsilon_1 \geq \underline{L} \\ 0 & \text{if } \underline{L} > \varepsilon_1. \end{cases} \tag{16}$$

The first region, $\varepsilon_1 \geq \bar{L}$, is the unconstrained region. In the intermediate region, for values of ε_1 between $\underline{L}$ and $\bar{L}$, the arbitrageur liquidates less than it would if unconstrained to keep the price high. This is the region in which price

[7] An arbitrageur with a longer horizon (as the one described in Section IV.B) displays a stronger willingness to buy the asset to meet the capital requirement.

manipulation allows it to meet the capital constraint: By inflating the price beyond the one that would prevail if the arbitrageur were unconstrained, each unit of position counts more toward the capital requirement.[8] Contrary to the case of a small initial position where the arbitrageur sells more than if it were not constrained, here it sells less and may even buy to support the price.

We solve for the optimal quantity of the strategic trader, x^*, by first obtaining the expected size of the arbitrageur's position after trading at $t = 1, E[\theta_1]$, and using this in (9) to obtain the result.

PROPOSITION 2: *Suppose that the arbitrageur holds a large initial position relative to market liquidity, $\theta_0 > \beta M$. Then, ordered in increasing levels of the arbitrageur's initial debt, B_0, we have the five regions.*

- *(i) In region I, where the probability that the arbitrageur is unconstrained is 1, the strategic trader does not trade, $x_I^* = 0$.*
- *(ii) In region II, where the probability that the constraint is binding and the probability that the arbitrageur is unconstrained are both greater than zero and add up to 1, the strategic trader sells the asset, $x_{II}^* < 0$.*
- *(iii) In region III, where (a) the constraint on the arbitrageur is binding with probability 1, the strategic trader sells the asset, $x_{IIIa}^* < 0$; or (b) the probabilities that the arbitrageur is unconstrained, is constrained, or chooses to fully liquidate are all greater than zero and add up to 1, and the strategic trader trades an amount that is either positive, negative, or zero, x_{IIIb}^*.*
- *(iv) In region IV, where the probability that the constraint is binding and the probability that the arbitrageur chooses to fully liquidate are both greater than zero and add up to 1, the strategic trader trades an amount that is either positive, negative or zero, x_{IV}^*.*
- *(v) In region V, where the probability that the arbitrageur chooses to fully liquidate is equal to 1, the strategic trader acquires $x_V^* = \frac{\theta_0}{4} > 0$.*

The expressions for $x_{II}^*, x_{IIIa}^*, x_{IIIb}^*, x_{IV}^*$, and the cutoff debt levels that separate the regions, $U_I, U_{II}, U_{III}, U_{IV}$, are presented in the Appendix.

The decisions by the strategic trader are analyzed in detail in the following corollaries.

COROLLARY 3: *Suppose that the arbitrageur holds a large initial position, $\theta_0 > \beta M$. Then, (i) while the probability of full liquidation by the arbitrageur is zero, the strategic trader's optimal quantity decreases with the arbitrageur's initial debt level, $\frac{\partial x^*}{\partial B_0} < 0$, and when the probability of full liquidation by the arbitrageur is greater than zero and smaller than 1, the strategic trader's optimal quantity everywhere increases with the arbitrageur's initial debt level, $\frac{\partial x^*}{\partial B_0} > 0$; and, (ii) for one level of the arbitrageur's initial debt, B_0, in regions in which the probability*

[8] Note that this is different from other models of price manipulation. In Allen and Gale (1992) and Allen and Gorton (1992), for example, manipulators trade to appear informed and to induce other market participants to trade.

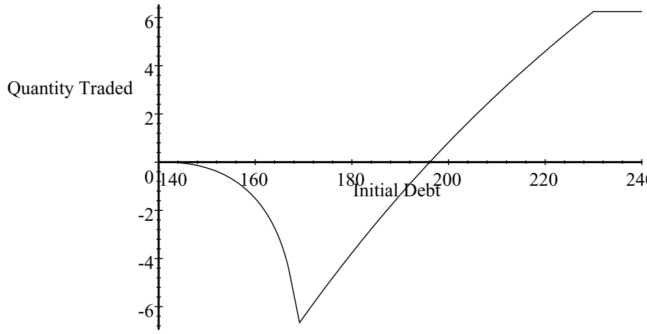

Figure 3. Strategic trader's optimal quantity for varying B_0; arbitrageur holds large position. Parameter values: initial position of the arbitrageur $\theta_0 = 25$, maximum absolute size of the liquidity shock $R = 5$, market depth $\beta = 5$, capital required per unit position $M = 2$, and fundamental value of the asset $I = 10$. This implies for the cutoff points of initial debt $U_I = 141.25, U_{II} = 167.22, U_{III} = 169.17$, and $U_{IV} = 230.0$.

of full liquidation is greater than zero and smaller than 1*, the strategic trader's optimal trading volume is zero.*

Corollary 3 shows that the strategic trader sells at date 1 when the arbitrageur's debt level is relatively low (region II), because doing so exploits the arbitrageur's trading to fulfill the capital requirement. When the arbitrageur is heavily indebted (region V), the strategic trader buys to take advantage of the anticipated immediate liquidation by the arbitrageur. Since the strategic trader's trading volume is continuous in the arbitrageur's debt level, this implies that the strategic trader does not trade for some initial debt level even when the arbitrageur's capital constraint is binding. The reason for this is that the strategic trader is uncertain whether the arbitrageur, after liquidity demand is realized, will liquidate immediately or will keep going for a period longer.

Figure 3, which plots the strategic optimal trading quantity as a function of the arbitrageur's initial debt level, illustrates that when the arbitrageur's initial position is large, the strategic trader remains inactive at sufficiently low levels of the arbitrageur's debt. If the arbitrageur's initial debt level is above a given threshold, the strategic trader sells, knowing that the arbitrageur is likely to keep the price high by liquidating only a small fraction of its position or even increasing it. When the arbitrageur is highly levered, the probability of complete liquidation is high enough to make the strategic trader buy the asset. The strategic trader profits from the pressure put on prices from a liquidating arbitrageur.

Figure 4 shows that the strategic trader's profitability varies significantly with the arbitrageur's initial debt. Given that the strategic trader's profits are positively related to its trading volume, Corollary 3 implies that the strategic trader's expected profit first rises, then falls, and finally rises again with the debt level, B_0, of a large arbitrageur. The strategic trader's expected profit

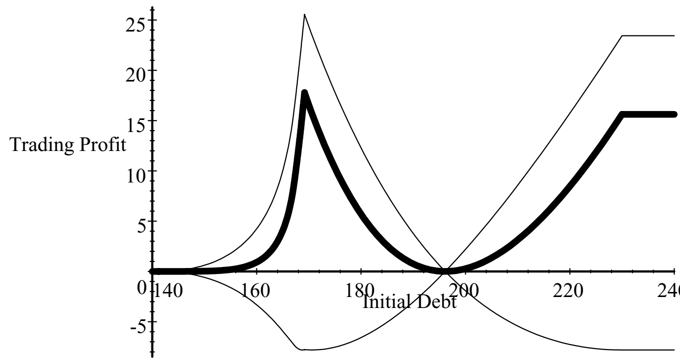

Figure 4. Strategic trader's expected profit for varying B_0; arbitrageur holds large position. The thick line plots the expected total profit $xE[(P_2 - P_1)]$. The line that is negative for low levels and positive for high levels of B_0 plots the expected first-period book profit $x(I - E[P_1])$, and the third line the expected second-period book profit $-x(I - E[P_2])$. Parameter values: Initial position of the arbitrageur $\theta_0 = 25$, maximum absolute size of the liquidity shock $R = 5$, market depth $\beta = 5$, capital required per unit position $M = 2$, and fundamental value of the asset $I = 10$. This implies for the cutoff points of initial debt $U_I = 141.25, U_{II} = 167.22, U_{III} = 169.17$, and $U_{IV} = 230.0$.

has two maxima, the first at the debt level at which the probability of a full liquidation is zero and the second for debt levels at which the arbitrageur always fully liquidates. At one particular B_0 between these levels, the strategic trader has zero trading profit. Denote this value of B_0 as $B_0(0)$.

COROLLARY 4: *Suppose that the arbitrageur holds a large initial position, $\theta_0 > \beta M$. Then, trading by the strategic trader (i) increases the probability of complete liquidation for $B_0 < B_0(0)$ and diminishes the probability of complete liquidation for $B_0 > B_0(0)$; and, (ii) diminishes the probability of price manipulation for $B_0 < B_0(0)$ and increases the probability of price manipulation for $B_0 > B_0(0)$.*

Figure 5 documents the result of Corollary 4 in a plot of the arbitrageur's probability of early liquidation both when strategic traders are present and when they are absent. The presence of the strategic trader lowers the cutoff debt level above which the arbitrageur finds full liquidation optimal. For a range of debt levels, the probability of full liquidation is already positive without a strategic trader but increases with its presence.

When the actions of strategic traders make it very expensive for the arbitrageur to support prices and meet the constraint, the arbitrageur decides, instead, to liquidate its entire position. As a result, strategic traders that profit at the expense of financially weak arbitrageurs trying to stay alive increase the likelihood that these arbitrageurs will fail. The actions of strategic traders thus create additional risks to arbitrage activities. Keynes once remarked that markets can be out of line for longer than one might stay solvent. It appears that markets can do even worse and precipitate the downfall of those that try

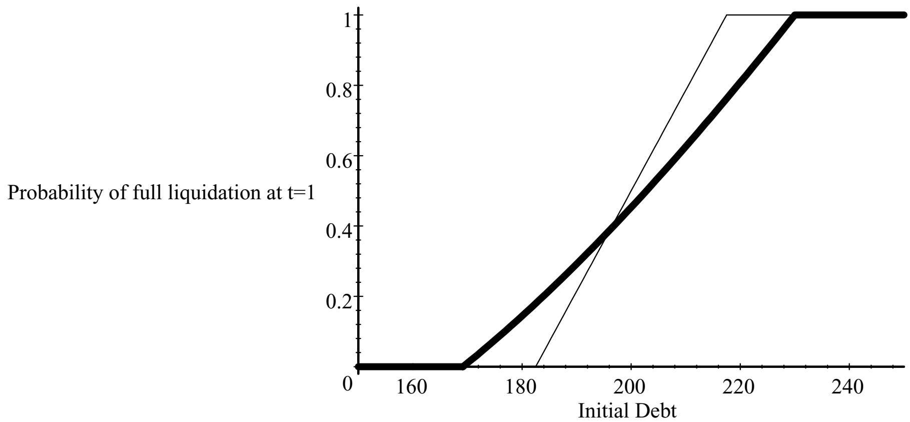

Figure 5. Probability of full arbitrageur liquidation at $t = 1$ without a strategic trader in the market (thin line) and with a strategic trader in the market (thick line) for varying B_0; arbitrageur holds large position. Parameter values: initial position of the arbitrageur $\theta_0 = 25$, maximum absolute size of the liquidity shock $R = 5$, market depth $\beta = 5$, capital required per unit position $M = 2$, and fundamental value of the asset $I = 10$. This implies for the cutoff points of initial debt $U_3 = 169.17$ and $U_4 = 230.0$.

to align prices with fundamentals, by riding along on their backs to the point of collapse. And the larger the trader, the more helpless it may be. To pull off his trick, John Rusnak, the foreign exchange arbitrageur who lost $700 million for Allied Irish Banks (AIB), had to deal with other traders that were well aware of his trades and of his weak financial condition.[9]

Note also, however, that the action of buying by the strategic trader when the arbitrageur's debt level is very high makes the strategy of supporting prices more attractive for the arbitrageur and reduces the probability that it might have to completely liquidate.

D. Implications of Strategic Trading for Prices and Welfare

In this section, we analyze how trading by the strategic trader affects prices as well as the profits earned by the arbitrageur. The analysis also enables us to assess the welfare effects from price distortions introduced by strategic trading. This is done by focusing on region IV, which, of all the regions, contains all the relevant consequences that capital requirements impose on traders in financial markets, namely, the need to meet the capital requirement, or, in the event that this is too expensive, immediate liquidation. To single out the effects of strategic trading, we compare a market with strategic trading with an otherwise identical market without strategic trading. To keep the analysis as simple as possible, we assume parameters that imply that the arbitrageur either is constrained or liquidates even when no strategic trader is present.

[9] See Plender (2002). Rusnak accumulated a position in currency forward much beyond AIB's regular currency positions and quickly became noticed by other traders, who kept trading with him.

When there is a possibility that the arbitrageur fully liquidates, the asset price is

$$P_1 = \begin{cases} I + \frac{\theta_0 \beta M - \beta(\theta_0 I - B_0) - \beta M \varepsilon_1}{\beta(\theta_0 - \beta M)} - \frac{\beta M x^*_{IV}}{\beta(\theta_0 - \beta M)} & \text{if } \varepsilon_1 \geq 2\frac{\beta M \theta_0 - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} - \frac{2\beta M x^*_{IV}}{\beta M + \theta_0} \\ I - \frac{\theta_0}{\beta} + \frac{\varepsilon_1}{\beta} + \frac{x^*_{IV}}{\beta} & \text{if } \varepsilon_1 < 2\frac{\beta M \theta_0 - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} - \frac{2\beta M x^*_{IV}}{\beta M + \theta_0}. \end{cases} \tag{17}$$

If the strategic trader sells the asset, for values of ε_1 such that the arbitrageur is constrained but does not fully liquidate, the price of the asset at date 1 is higher in the presence of the strategic trader. By selling, the strategic trader forces the arbitrageur to manipulate the prices more to meet the constraint. Adding just the strategic trader's amount to its order is not sufficient for the arbitrageur to satisfy the constraint, because the overall level of required risk capital increases as the arbitrageur must hold larger positions as a result of the actions of the strategic trader.[10] For values of ε_1 such that the arbitrageur must fully liquidate, the price of the asset is lower in the presence of the strategic trader, since both these traders exert downward pressure on the price. The cutoff level of ε_1 below which the arbitrageur fully liquidates is higher with the strategic trader present.

At time 2, the price is

$$P_2 = \begin{cases} I - \frac{\theta_0\left(\theta_0 - x^*_{IV} - \varepsilon_1\right) - \beta(\theta_0 I - B_0)}{\beta(\theta_0 - \beta M)} + \frac{\varepsilon_2}{\beta} - \frac{x^*_{IV}}{\beta} & \text{if } \varepsilon_1 \geq 2\frac{\beta M \theta_0 - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} - \frac{2\beta M x^*_{IV}}{\beta M + \theta_0} \\ I + \frac{\varepsilon_2}{\beta} - \frac{x^*_{IV}}{\beta} & \text{if } \varepsilon_1 < 2\frac{\beta M \theta_0 - \beta(\theta_0 I - B_0)}{\beta M + \theta_0} - \frac{2\beta M x^*_{IV}}{\beta M + \theta_0}. \end{cases}$$

Since the aggregate trades of the arbitrageur and the strategic trader across the two time periods are constant, the results at time 2 reverse those in period 1: When the strategic trader buys the asset at time 2 and the arbitrageur has not fully liquidated at time 1, the price at time 2 is higher than if no strategic trader were present. The same holds if the arbitrageur has liquidated its entire position at time 1. The preceding results are reversed if the strategic trader instead buys the asset at time 1 and sells it at time 2.

The price distortions created by the presence of the strategic trader have welfare implications for the other traders in the market. Accurate and less volatile prices are of special importance to liquidity traders that need to trade for consumption and hedging reasons. The following corollary states the effects on prices from strategic trading.

COROLLARY 5: *Suppose the arbitrageur is financially constrained with positive probability and may fully liquidate prematurely (region IV).*

(a) If the strategic trader sells (buys) at time 1, strategic trading increases (decreases) the range of possible market prices at time 1.

[10] This can be seen when rewriting inequality (5): $\theta_{t-1}(I + \frac{\varepsilon_t}{\beta}) - B_{t-1} \geq \theta_t M - \theta_{t-1}(\frac{\theta_t - \theta_{t-1}}{\beta} - \frac{x_t}{\beta})$. Nullifying the strategic trader's supply only keeps the second term of the right-hand side of the inequality constant. The first term increases, requiring additional position buildup to meet the capital constraint.

(b) *In the parameter range in which strategic trading does not affect whether there is full liquidation and the strategic trader sells (buys) at time 1, strategic trading increases (decreases) the deviation from the price that would prevail if the arbitrageur were unconstrained in both periods.*

In the presence of a financially healthy arbitrageur, the price is the one that prevails in a well-functioning market. A financially constrained arbitrageur finds it more difficult to execute a smooth liquidation of its holdings, and this distorts prices. The presence of a selling strategic trader disrupts the liquidation process and distorts prices even more. The greater distortion created by the strategic trader can be seen by contrasting the thick line in Figure 6 (the prices with a constrained arbitrageur and a strategic trader), which is always farther away from the dashed line (the prices with an unconstrained arbitrageur) than the thin line (the prices with a constrained arbitrageur and no strategic trader). This distortion is especially harmful if the arbitrageur collapses and at the same time the strategic trader is selling, because with no one to absorb the liquidity demand, buying liquidity traders trade at an extremely unfavorable price. Furthermore, the presence of the strategic trader increases the probability that this occurs. These results hold also at time 2. Consider, for example, a low liquidity demand at time 1 such that a constrained arbitrageur liquidates its entire position at that date. Whereas an unconstrained arbitrageur liquidates the remainder of its position at time 2, a constrained trader remains

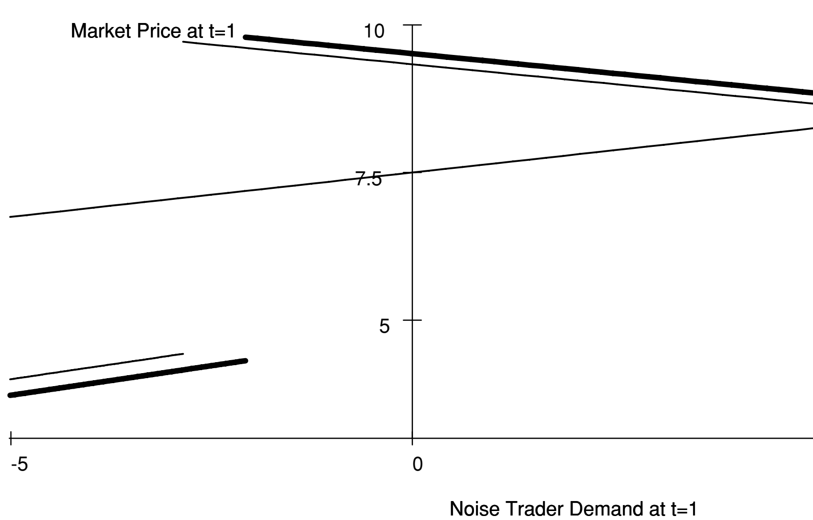

Figure 6. Price level of the asset at time 1 in region IV as a function of liquidity demand; arbitrageur holds large position. The thin line in the middle depicts the price level for an unconstrained arbitrageur. The thick line characterizes the price level for a constrained arbitrageur when a strategic trader is present and the thin outer line shows the price level in the absence of a strategic trader. Parameter values: initial position of the arbitrageur $\theta_0 = 25$, maximum absolute size of the liquidity shock $R = 5$, market depth $\beta = 5$, capital required per unit position $M = 2$, and fundamental value of the asset $I = 10$.

inactive, which implies a higher price in the latter case. A strategic trader that sells the asset at date 1 buys it back at time 2, which leads to a positive price distortion caused by the capital requirement. The presence of a strategic trader that sells the asset at time 1 biases the price further upward, because it reverses its trade at time 2.

The range of possible prices measures the uncertainty a liquidity trader faces when participating in the market. Liquidity trading that is driven by hedging objectives is negatively affected by higher uncertainty. Given that the actions of the strategic trader increase this uncertainty at time 1, they are not welfare improving from the perspective of these liquidity traders. The larger range in the presence of strategic trading at time 1 can be seen in Figure 6. At time 2, the range of possible prices is independent of the presence of strategic trading. The reason for this is that in cases both with and without strategic trading, trading by the arbitrageur does not vary with liquidity demand at date 2.

The results described previously are reversed when the strategic trader buys the asset at time 1, because in this event, the strategic trader provides liquidity to a fragile arbitrageur and therefore contributes to a smoother liquidation process.

The effects on price resulting from the strategic trader's actions show that it contributes to an increase in the range of observed prices when it sells the asset and reduces the range of prices when it buys the asset. Strategic selling thus increases the volatility of prices. An arbitrageur that attempts to stabilize prices because it cannot trade in the other direction and liquidate as much as it would like in a liquid market finds it more costly to keep the price high in the presence of a strategic trader that sells the asset. The strategic trader forces the arbitrageur to maintain a larger position in the asset to compensate for the additional supply. Buying at a high price from a strategic trader that counts on prices being artificially manipulated by an arbitrageur that is fighting for its survival can be very costly and reduces the ex ante probability of price manipulation. Thus, the range of possible price increases when a strategic trader is present and is willing to sell to a constrained arbitrageur that is forced to buy more simply because it cannot sell due to market thinness. In sum, this paper provides an additional argument for the existence of "destabilizing speculation" in financial markets (see, e.g., Hart and Kreps (1986) and DeLong et al. (1990)).

III. Lending and Strategic Trading When the Arbitrageur Is Financially Constrained

Can it be optimal for the strategic trader to loosen the constraint on the arbitrageur to generate higher trading profits? We address this question by analyzing the strategic trader's incentives to lend to the arbitrageur without imposing a capital requirement on the loan. The loan is made before trading occurs at $t = 1$. Lending without requiring risk capital allows the arbitrageur to replace existing debt that is subject to a capital requirement with new debt that is not. Alternatively, it allows the arbitrageur to finance the purchase of additional assets with new debt that does not carry a capital requirement.[11] In both cases, while the total amount owed is still B_0, for the purpose of computing the capital requirement it is as if the debt level were at B_0 minus the amount lent by the strategic trader.

To present the arguments in the simplest possible way, we maintain the assumptions that credit does not carry interest and that the loan is riskless.[12] These assumptions imply that the profit of the strategic trader that lends and does not trade is zero, and therefore enable us to study the incentives for the strategic trader to lend purely for trading considerations rather than to earn profits directly from lending. In addition, we assume that the strategic trader does not lend if it is indifferent between lending and not lending.

Because a loan by the strategic trader relaxes the arbitrageur's capital constraint, it is optimal for the strategic trader to lend only if it benefits its trading. This is not immediately obvious, because a financially healthier arbitrageur seems to provide fewer opportunities for the strategic trader to profit. Also, although a loan by the strategic trader reduces the arbitrageur's capital requirement, it is not necessarily the case that accepting the loan increases the arbitrageur's wealth. The reason for this is that the resulting change in the strategic trader's behavior potentially increases the arbitrageur's cost of liquidating its position. As the following proposition states, however, there are instances when both parties benefit from lending.

PROPOSITION 3:

(i) When the arbitrageur holds a small position, the strategic trader does not lend to the arbitrageur.

(ii) When the arbitrageur holds a large position, there exists a nonempty set of values of the initial debt level, B_0, for which the strategic trader offers to lend to the arbitrageur and the arbitrageur accepts the offer.

Lending by the strategic trader leads to the same behavior on the part of the arbitrageur as reducing its debt level by the amount lent. As Figure 2 demonstrates, for an arbitrageur with a small initial position, the strategic trader's profit never increases with a decline in the arbitrageur's debt level. Thus, the strategic trader does not lend to an arbitrageur holding a small position.

When the arbitrageur holds a large position, the strategic trader's expected profit is not monotonic in the arbitrageur's debt level (see Figure 4). Consider, for example, $B_0 = B_0(0)$. Then, the strategic trader finds it optimal to not trade, because it faces considerable uncertainty as to whether the arbitrageur trades to meet the capital requirement or liquidates its entire position. The resulting inactivity yields it a zero profit from trading. In this situation, lending even a relatively small amount reduces the uncertainty regarding the arbitrageur's course of action and makes the strategic trader profit from trading.

[11] If the strategic trader is already a creditor to the arbitrageur, lending without a capital requirement is identical to waiving the capital requirement for a portion or the entire stock of existing loans.

[12] We discuss later the situation in which the loan is risky.

Note that lending does not reduce the level of uncertainty about liquidity demand; rather, it simply makes the arbitrageur's response to liquidity trading more predictable.

For the purpose of lending, in general it is not the source of uncertainty that matters but the presence of uncertainty in and of itself. For example, a strategic trader that observed the demand by noise traders but was uncertain about the arbitrageur's indebtedness, B_0, would also have an incentive to lend with the objective of trading against the arbitrageur. Such a strategic trader would sometimes be uncertain as to whether the arbitrageur completely liquidates or trades to meet the constraint. Knowing the liquidity shock, however, the strategic trader would also profit from noise trading.

Despite relaxing the capital constraint, lending may have a negative effect on the arbitrageur's profits because the strategic trader alters its trading. There are, however, always initial debt levels for which the arbitrageur accepts credit offers for certain amounts. Suppose again that $B_0 = B_0(0)$. Then, the arbitrageur's profits increase in a discrete way if a loan by the strategic trader makes it unconstrained because its capital constraint is relaxed and the strategic trader's demand remains unchanged at zero. Given the arbitrageur's increase in profits from lending by the strategic trader, it will also accept a somewhat smaller loan even if this implies that the capital constraint is binding for some values of liquidity demand. Therefore, levels of initial debt exist for which lending is profitable for both the arbitrageur and the strategic trader. This argument is valid not only when $B_0 = B_0(0)$, but also for values of initial debt between the one that defines the maximum trading profits when the strategic trader sells and $B_0(0)$ (see Figure 4). In this region and even for some initial debt levels beyond $B_0(0)$, it is always possible to find a loan amount that improves the expected profits of the arbitrageur in a discrete way without decreasing the strategic trader's profits. The continuity of the profit functions ensures that a mutually strictly beneficial loan amount can be found.

The preceding example also illustrates that the arbitrageur does not benefit only from the presence of a strategic trader that provides liquidity. The arbitrageur may be better off even when the strategic trader trades against it, provided that a loan obtained outweighs the negative effects of the strategic trader's trading activity.

In our analysis we assume that the loan extended by the strategic trader is riskless. As mentioned before, this is often not the case. However, even though the loan is risky, as long as the expected loss from lending is small, the strategic trader is willing to extend credit. The expected increase in trading profits justifies granting a subsidized loan to the arbitrageur. For the strategic trader's decision to offer credit, the necessary size of the loan matters. If the arbitrageur has an initial debt level above $B_0(0)$, the strategic trader needs to make a larger loan to achieve the same level of trading profits than if the arbitrageur's initial debt level is below $B_0(0)$. Even if the per unit expected loss is the same in both cases, the larger loan will result in a larger overall loss. Consequently, the strategic trader will decline to lend for $B_0 > B_0(0)$ first when the loan is risky.

In certain situations the strategic trader has an incentive to undertake actions that are equivalent to increasing rather than decreasing the arbitrageur's debt level.[13] Consider, for example, the case of $B_0 > B_0(0)$. An increase in the debt level increases the likelihood of immediate liquidation by the arbitrageur, which the strategic trader is able to exploit to its own advantage. If it is already a lender to the arbitrageur, the strategic trader can attempt to achieve this by suddenly changing the rules, for example, by increasing the capital requirement on the loans. This course of action is effective if the arbitrageur has difficulties replacing the strategic trader's loans with loans from other creditors with better terms. Thus, our model is also consistent with what has sometimes been referred to as "pulling the plug," or the strategic increase in collateral required to force liquidation. Among practitioners, there is the view that when markets spot a loser, they will often ride the market further in that direction until the collateral is almost exhausted, and then increase collateral requirements, forcing the arbitrageur to unwind its positions precipitously.

IV. Extensions

We present three extensions to the basic framework. The purpose of these extensions is to provide additional discussion of the results obtained and to show that the results presented previously can, at the cost of greater analytical complexity, be replicated under more general conditions.

A. General Capital Constraints for the Arbitrageur

The ability of the strategic trader to make profits depends on the predictability of price deviations caused by a fragile arbitrageur trying to meet the binding constraint. Until now we have considered capital constraints similar to those explicitly imposed by regulatory agencies or from in-house risk management systems, as well as constraints implicitly imposed by lenders and counterparties in transactions with low credit rating and insufficiently capitalized traders. The particular type of constraint the arbitrageur faces is, however, of secondary importance when compared to the presence of the constraint itself. For example, consider a capital constraint that is impossible to meet when the arbitrageur tries to manipulate prices. Suppose the per unit capital required is a function of the traders' positions, that is, capital equals $\mu(\theta_t)$ per unit, instead of the constant M considered before. For $\theta_t > 0$, this gives $\theta_t P_t - B_t \geq \theta_t \mu(\theta_t)$, which yields

$$\theta_{t-1}\left(I - \frac{\theta_{t-1}}{\beta} + \frac{x_t + \varepsilon_t}{\beta}\right) - B_{t-1} \geq \theta_t\left(\mu(\theta_t) - \frac{\theta_{t-1}}{\beta}\right). \tag{18}$$

[13] This is a risk that is acknowledged among traders. The following statement was made by Jeffrey Grundlach, a portfolio manager at Trust Co. of the West in describing the failure of hedge funds investing in mortgage backed assets run by David Askin: "[I]t isn't a good idea to leverage up volatile securities when dealers can mark down securities and force margin calls any time they want." See "Cracks Appear in Granite Partners Fund," *Wall Street Journal*, March 30, 1994.

For $\mu(\theta_t) = M + N\theta_t$ with $M > 0$ and $N \geq \frac{1}{\beta}$, the arbitrageur can no longer meet the financial constraint by trading to attempt to support prices. All positions are then small positions, as defined earlier. Although this type of constraint is theoretically possible, it would be very hard to implement. It would require the knowledge of both the condition of the market, β, as well as the financial situation of the arbitrageur at all times. If, for example, the market depth fluctuates randomly, then for values of $\frac{1}{\beta} > N \geq 0$, the arbitrageur would be able to meet the financial constraint by supporting prices. A different type of constraint is given by moving the time subscripts in (18) forward by one period and setting the right-hand side to zero, giving

$$\theta_t \left(I - \frac{\theta_t}{\beta} + \frac{x_{t+1} + z_\alpha}{\beta} \right) - B_t \geq 0, \tag{19}$$

where z_α is such that the probability of the liquidity shock being more adverse than z_α is equal to α, $\Pr(\varepsilon_{t+1} < z_\alpha) = \alpha$. This is equivalent to a forward-looking Value-at-Risk (VaR) measure. Alternatively, it is equivalent to a credit rating-based constraint on θ_t that automatically adjusts for market illiquidity, and which provides a confidence level of $1 - \alpha$ to lenders and trading counterparties that the arbitrageur will be able to meet its debt obligations. In this case the previous results for the case of small positions apply, but the arbitrageur cannot meet the constraint by providing price support. The only way the arbitrageur could meet the constraint by manipulating prices is if the computation underestimated the factor necessary to account for market illiquidity. In practice such underestimation could easily occur. To implement the capital constraint with a VaR system, or with a credit rating-based constraint, the history of price changes up to the current period is what matters. This history depends on two pieces of information, namely, the series of shocks drawn and the evolution of both the arbitrageur's positions and the level of indebtedness. Suppose, for example, that the shocks to prices had caused a buildup in the arbitrageur's position, as well as in its level of indebtedness, which is what would happen if the sequence of past shocks had a negative average. As the arbitrageur moved toward the constraint, its ability to absorb negative shocks would be progressively weakened. The fragility would be reflected in price changes with an increasingly negative conditional mean and a higher conditional variance. Volatility estimated using historical prices would then underestimate future volatility. This would be the same as incorrectly estimating β, the coefficient for market depth, thereby causing lenders and counterparties to incorrectly set N in $\mu(\theta_t)$ in (18) too low and opening up the opportunity for an arbitrageur with a large position to affect prices in order to meet the constraint.

B. *Arbitrageur with a Longer Horizon*

Our results can be extended to the case of an arbitrageur maximizing expected terminal wealth at a longer horizon of $2 < T < \infty$ periods. To maintain comparability, assume that $\theta_T = 0$. For a long horizon, T, this assumption will have a negligible impact on the arbitrageur's trade at $t = 1$. Let $W(\theta_0, B_0, T)$

be the optimized expected wealth of the arbitrageur with an initial position $\theta_0 > \beta M$ and debt level B_0. With T periods, the arbitrageur's objective function is

$$W(\theta_0, B_0, T) = \max_{\theta_1}\{(\theta_1 - \theta_0)(I - P_1) + E[W(\theta_1, B_1, T-1) \mid \varepsilon_1]\}. \tag{20}$$

The arbitrageur's capital constraint in the current period requires that

$$\theta_1 P_1 - B_1 \geq |\theta_1| M. \tag{21}$$

The effect of the constraint on future periods shows up through $W(\theta_1, B_1, T-1)$. Depending on the value of ε_1, there are four cases: (i) The arbitrageur trades unaffected by the constraint, $\frac{\partial E[W(\theta_1, B_1, T-1) \mid \varepsilon_1]}{\partial \theta_1} = -\frac{\theta_0}{T}$, giving $\theta_1^* - (\frac{T-1}{T})\theta_0 + \frac{x+\varepsilon_1}{2} = 0$; (ii) the constraint is not currently binding, but because it might bind in the future it affects the arbitrageur's immediate trades, $\frac{\partial E[W(\theta_1, B_1, T-1) \mid \varepsilon_1]}{\partial \theta_1} < -\frac{\theta_0}{T}$ for some ε_1, and for those ε_1 the value θ_1^* solves $\theta_1^* - \theta_0 + \frac{x+\varepsilon_1}{2} - \frac{\partial E[W(\theta_1, B_1, T-1) \mid \varepsilon_1]}{\partial \theta_1} = 0$; (iii) the constraint is binding in the current period, $\theta_1^* = \theta_C^*$; and, (iv) the arbitrageur liquidates fully at $t = 1, \theta_1^* = 0$. The equality $\frac{\partial E[W(\theta_1, B_1, T-1) \mid \varepsilon_1]}{\partial \theta_1} = -\frac{\theta_0}{T}$ results from the requirement that at the terminal period $T, \theta_T = 0$. Cases (i), (iii), and (iv) are those described in Section II.C.2. Case (ii) is the only new scenario. For the remainder of the discussion, we denote by $\theta_{1,k}^*$ the θ_1^* in case k. To solve for x^*, we need to determine both $E[\theta_1^*]$ and $E[\theta_2^*]$ and then plug these into the first-order condition $\theta_0 - 2E[\theta_1^*] - E[\theta_2^*] - 4x^* = 0$. We first obtain the cutoffs for the regions. Let $\bar{\mathcal{L}}_1$, $\bar{\underline{\mathcal{L}}}_1$, and $\underline{\mathcal{L}}_1$ be the cutoff points at $t = 1$ between cases (i) and (ii), (ii) and (iii), and (iii) and (iv), respectively. As before, $\underline{\mathcal{L}}_1$ is obtained from the value of ε_1 at which the arbitrageur is indifferent between immediate liquidation and buying the asset to support prices and meet the financial constraint. The cost of immediate liquidation is the same as in the previous section. The gains to the arbitrageur if it supports prices and survives are greater, however, in a T-period horizon than with two periods. Therefore, the arbitrageur is willing to bear greater losses immediately to ensure that the constraint is met and that it can continue to trade. This implies that $\underline{\mathcal{L}}_1 < \underline{L}$, or, that a much more severe shock is needed for the arbitrageur to fully liquidate immediately in a T-period horizon. The cutoff level $\bar{\underline{\mathcal{L}}}_1$ is determined by equating θ_1^C to $\theta_{1,(ii)}^*$. For all ε_1, $\theta_{1,(ii)}^* < \theta_{1,(i)}^*$ and $\frac{\partial E[W(\theta_1, B_1, T-1) \mid \varepsilon_1]}{\partial \theta_1} < -\frac{\theta_0}{T}$ indicate that the arbitrageur values the flexibility that small positions afford it. This causes $\bar{\underline{\mathcal{L}}}_1 > \bar{L}$. If T or R are large, the arbitrageur is unlikely to ever be unaffected by the constraint. This implies that $\bar{\mathcal{L}}_1 > R$, and allows us to ignore case (i). At $t = 2$, there are again four cases, and the analysis of the θ_2^* values and the cutoff levels are very similar. For a given θ_0, consider a debt level such that $\bar{\mathcal{L}}_1 > R > -R > \bar{\underline{\mathcal{L}}}_1$ and $\bar{\mathcal{L}}_2 > R > -R > \bar{\underline{\mathcal{L}}}_2$, implying that the constraint does not bind at $t = 1$ nor at $t = 2$. Since the arbitrageur enjoys maintaining financial flexibility, the arbitrageur will adjust its position downward at both dates. On average, the adjustment will occur more at $t = 1$ than at $t = 2$. This means that $\theta_0 - 2E[\theta_1^*] - E[\theta_2^*] > 0$, and the strategic trader will optimally buy the asset, $x^* > 0$. For a higher level of debt, such that $\bar{\underline{\mathcal{L}}}_1 > R > -R > \underline{\mathcal{L}}_1$ and

$R > \bar{\mathcal{L}}_2 > \underline{\mathcal{L}}_2 > -R$, the constraint is binding at $t = 1$, while at $t = 2$ there is some likelihood that the constraint will not bind, some likelihood that it will continue to bind in the future, and some likelihood of immediate liquidation. An unconstrained arbitrageur will trade so that $\theta_0 - E[\theta_1^*] = E[\theta_1^*] - E[\theta_2^*] = \frac{\theta_0}{T}$. For a constrained arbitrageur, we have $\theta_0 - E[\theta_1^*] < \frac{\theta_0}{T}$. At $t = 2$, if the area in which the constraint binds is smaller than the other two areas, the arbitrageur will liquidate a fraction of its holdings, giving $E[\theta_1^*] - E[\theta_2^*] > \frac{\theta_0}{T}$. In turn, this yields $\theta_0 - 2E[\theta_1^*] - E[\theta_2^*] < 0$, and the optimal trading for the strategic trader is to sell the asset, $x^* < 0$. Finally, when the debt level is so high that $\underline{L} > R$, the arbitrageur always liquidates at $t = 1$, and the strategic trader trades an amount equal to $x^* = \frac{\theta_0}{4}$. The more periods the constrained arbitrageur trades, the greater the range of initial conditions that give the strategic trader profits from actively trading. This is true even though a longer horizon gives the arbitrageur greater flexibility to manage its position. The strategic trader's profits are likely to be small when it trades against a large arbitrageur that faces a constraint that is not immediately binding. A fixed cost of trading can cause the strategic trader to refrain from trading against such an arbitrageur. When the arbitrageur faces a binding financial constraint or when there is a significant likelihood that it will liquidate its position and exit the market, the strategic trader has a greater incentive to trade and the results are again those in the previous section. Thus, extending the horizon alone does not change the results much.

C. A Market with Several Strategic Traders

So far we have assumed that the arbitrageur faces the actions of a single strategic trader. In reality, however, several, possibly many, traders are aware of the financial condition of a visible arbitrageur. Some lenders to the arbitrageur may decide to become strategic traders once they realize the weakened situation of the arbitrageur. However, the arbitrageur can also influence the number of strategic traders that are in the market. For instance, the arbitrageur can choose to borrow funds from less sophisticated institutions that do not trade, or that are prevented from trading the same asset as the arbitrageur by regulatory restrictions. More generally, the arbitrageur can try to select from whom it gets funding for trading or to whom it reveals information on its positions. Interestingly, small financially constrained arbitrageurs always find it better to attract more strategic traders to the market. The intuition for this is simple. An arbitrageur with a small position always sells the asset to meet the constraint. Strategic traders buy the asset and provide liquidity to the arbitrageur, thus cushioning the price impact of its trades and reducing the cost of meeting the constraint.

Formally this can be seen in an optimization problem of the strategic trader and the arbitrageur. In the case of n strategic traders, each trading x units at $t = 1$, and a single arbitrageur, an individual strategic trader's objective function that maximizes expected trading profit, defined as $E[x(P_2 - P_1)]$, is given as

$$\max_x \frac{x}{\beta}\left(\theta_0 - 2E[\theta_1] - 2(n-1)x^* - 2x\right). \tag{22}$$

The optimal size of the trade solves the first-order condition $\theta_0 - 2E[\theta_1(x^*)] - 2(n+1)x^* = 0$. The arbitrageur's objective function is

$$\max_{\theta_1} \theta_0 I - B_0 - (\theta_1 - \theta_0)\left(\frac{\theta_1 - \theta_0}{\beta} + \frac{nx + \varepsilon_1}{\beta}\right) - \theta_1\left(\frac{\theta_1}{\beta} + \frac{nx}{\beta}\right), \tag{23}$$

which has a first-order condition of $\theta_1^U = \frac{\theta_0}{2} - \frac{nx}{2} - \frac{\varepsilon_1}{4}$. For an arbitrageur with a small initial position, $\beta M > \theta_0 > 0$, the capital constraint requires that $\theta_1^C \leq \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - nx - \varepsilon_1)}{(\beta M - \theta_0)}$. The capital constraint is binding when

$$\bar{l}^n \equiv 2\frac{(\theta_0 + \beta M)(\theta_0 - nx) - 2\beta(\theta_0 I - B_0)}{(3\theta_0 + \beta M)} > \varepsilon_1. \tag{24}$$

For $\bar{l}^n < -R$, the arbitrageur's constraint is never binding, and therefore $x^* = 0$. The debt level below which the constraint on the arbitrageur is never binding remains the same as in the case of a single strategic trader. For a debt level such that the constraint is binding, the arbitrageur liquidates some of its position to meet the constraint, and the strategic trader buys the asset. This action by the strategic trader helps to reduce the cost to the arbitrageur of meeting the financial constraint. Also, from examining the constraint, for $x^* > 0$, the more strategic traders there are, that is, the higher the n, the more the constraint is relaxed. Finally, if we plug θ_1^U into the arbitrageur's objective function, we get $\theta_0 I - B_0 + \frac{(\varepsilon_1 + 2nx)^2 + 4\theta_0\varepsilon_1 - 4\theta_0^2}{8\beta}$, which also increases with the number of strategic traders. Thus, for a constrained arbitrageur with a small position in the asset, it is always better to face a greater number of strategic traders.

For a large arbitrageur the situation is not so straightforward. The constraint is $\theta_1^C \geq \frac{\theta_0(\theta_0 - nx - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)}$, so the arbitrageur finds it optimal to fully liquidate at $t = 1$ if

$$\underline{L}^n \equiv 2\frac{\beta M(\theta_0 - nx) - \beta(\theta_0 I - B_0)}{(\theta_0 + \beta M)} > \varepsilon_1. \tag{25}$$

Clearly, for $\underline{L}^n > R$, the arbitrageur always liquidates at $t = 1$ and the optimal action by the strategic trader is to buy $x^* = \frac{\theta_0}{2(n+1)}$. The level of debt for which the arbitrageur is always forced to liquidate is $\theta_0 I + \frac{R(\theta_0 + \beta M)}{2\beta} - \frac{\beta M(n+2)\theta_0}{2\beta(n+1)}$, which increases in n. When an arbitrageur with a long position faces a high likelihood of immediate forced liquidation, strategic traders that buy help reduce the cost of liquidation. It is then better to have more strategic traders active in the market. When the large arbitrageur's level of indebtedness is intermediate between these two levels, and $\bar{L}^n \geq R > -R > \underline{L}^n$, we have

$$E[\theta_1] = \frac{\theta_0(\theta_0 - nx) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)}. \tag{26}$$

In this intermediate region, we showed above that the optimal amount traded by a single strategic trader was $x^* = \frac{2\beta(\theta_0 I - B_0) - \theta_0(\theta_0 + \beta M)}{2(\theta_0 - (n+1)\beta M)}$. As a group, the maximum quantity that the strategic traders trade in this region is $-n(\frac{\theta_0}{2} - \frac{R(\theta_0 + \beta M)}{2(\theta_0 - \beta M)})$ when $B_0 = \theta_0 I - \frac{\beta M(2+n)\theta_0}{2\beta} - R\frac{(\theta_0 + \beta M)(\theta_0 - (n+1)\beta M)}{2\beta(\theta_0 - \beta M)}$. Also, when $E[\theta_1(0)] = \frac{\theta_0}{2}$, the strategic trader abstains from trading, $x^* = 0$, and ${B^n}_0(0)$ solves

$$\frac{(R - \underline{L}^n(x=0))\big(2\big(\theta_0^2 - \beta(\theta_0 I - B_0)\big) - (R + \underline{L}^n(x=0))\theta_0\big)}{2R\theta_0(\theta_0 - \beta M)} = 1. \tag{27}$$

This expression does not depend on n. Recall that for $B_0 < B_0(0)$, strategic traders sell the asset. Then, an increase in the number of strategic traders increases the cost to the arbitrageur of buying to support prices and meet the constraint. The higher cost increases the probability of full liquidation by the arbitrageur at $t = 1$. Conversely, for $B_0 > B_0(0)$, strategic traders buy the asset and reduce the cost and the likelihood of full liquidation by the arbitrageur at $t = 1$. In summary, for $B_0 < B_0(0)$, it is better that the arbitrageur deals with fewer strategic traders, whereas for $B_0 > B_0(0)$, the arbitrageur prefers to face more strategic traders. If the arbitrageur has means at its disposal to influence the number of strategic traders, it will use them to its own advantage.

V. Conclusion

This paper presents an analysis of trading in markets when large and prominent arbitrageurs do have access to limited amounts of capital. We show that it is not enough to study the trading activity of these traders in isolation, as there are opportunities for other market participants to exploit arbitrageurs' financial constraints. When an arbitrageur's financial flexibility is limited, its trades, and thus market prices, become predictable. Financial fragility may lead to either excessive liquidation or excessive position holding. Excessively large positions occur when by manipulating prices it is possible to avoid violating capital constraints. Trading based on financial constraints of other traders has implications for the functioning of financial markets. As long as the positions of financially fragile arbitrageurs are not too large, such strategic trading reduces excessive liquidation and smoothes price fluctuations. If the positions of a fragile arbitrageur are large, however, the effect is reversed if the demise of the arbitrageur is not yet imminent. In these situations, strategic trading itself increases the probability of the fragile trader's failure. Strategic traders take advantage of depressed prices during the time of the arbitrageur's collapse, as well as exploit price manipulation efforts by the fragile arbitrageur before its collapse. Strategic traders may decide to lend to a financially fragile arbitrageur to allow it to continue trading. The inclusion of corporate finance issues into the analysis of capital markets provides novel insights into the behavior of traders. Through our examination of the strategies of traders that attack an arbitrageur until they collapse, we shed light on the risks that arbitrageurs face in financial markets.

Appendix: Proofs

Proof of Proposition 1: To solve the strategic trader's problem, we proceed as follows: (i) Using $\theta_1(x, \varepsilon_1)$, we compute $E[\theta_1(x, \varepsilon_1)]$; and, (ii) we use $E[\theta_1(x, \varepsilon_1)]$ in the strategic trader's first-order condition to obtain x^* as the solution to $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$. We then plug the optimal x^* into $\bar{l}$ and $\underline{l}$ to solve for the boundaries for the regions.

We repeat (13) here for convenience:

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if} \quad \varepsilon_1 \geq \bar{l} \\ \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x - \varepsilon_1)}{(\beta M - \theta_0)} & \text{if} \quad \bar{l} > \varepsilon_1 \geq \underline{l} \\ 0 & \text{if} \quad \underline{l} > \varepsilon_1, \end{cases}$$

where $\bar{l} \equiv 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)}$ and $\underline{l} \equiv \frac{\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\theta_0}$.

In computing $E[\theta_1]$ we need to consider five regions:

(i) Region I: $-R > \bar{l}$, where the arbitrageur is always unconstrained. In this case, $E[\theta_1] = \frac{1}{2}(\theta_0 - x)$ and $x_I^* = 0$.
(ii) Region II: $R > \bar{l} > -R > \underline{l}$, where the arbitrageur is sometimes constrained and otherwise unconstrained.
(iii) Region III: (a) $\bar{l} > R > -R > \underline{l}$, where the arbitrageur is always constrained; or, (b) $R > \bar{l} > \underline{l} > -R$, where the arbitrageur is unconstrained, constrained, or forced to fully liquidate.
(iv) Region IV: $\bar{l} > R > \underline{l} > -R$, where the arbitrageur is sometimes constrained and otherwise is forced to liquidate.
(v) Region V: $\underline{l} > R$, where the arbitrageur is always forced to completely liquidate. In this case, $E[\theta_1] = 0$ and $x_V^* = \frac{\theta_0}{4}$.

For (ii), where $R > \bar{l} > -R > \underline{l}$, we have

$$E[\theta_1] = \frac{(\theta_0 - x)(\beta M - 3\theta_0) + 2\beta(\theta_0 I - B_0)}{4(\beta M - \theta_0)} - \frac{(\beta M + 3\theta_0)}{4(\beta M - \theta_0)}\left(\frac{R^2 + \bar{l}^2}{4R}\right),$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\frac{(\beta M + 3\theta_0)}{8R(\beta M - \theta_0)} \times (R + \bar{l}(x^*))^2 - 3x^* = 0$, with the solution

$$x_{II}^* = \theta_0 - \frac{(5\theta_0 - 7M\beta)(M\beta + 3\theta_0)R}{2(M\beta + \theta_0)^2} - \frac{2(I\theta_0 - B_0)\beta}{(M\beta + \theta_0)}$$
$$- \frac{\sqrt{6R(\beta M + 3\theta_0)(\beta M - \theta_0)\Big((\theta_0 + \beta M)^2\theta_0 + R(2\beta M - \theta_0)(\beta M + 3\theta_0) - 2\beta(\theta_0 I - B_0)(\theta_0 + \beta M)\Big)}}{(\beta M + \theta_0)^2}. \tag{A1}$$

For (iii) (a), where $\bar{l} > R > -R > \underline{l}$, we have

$$E[\theta_1] = -\frac{\theta_0}{(\beta M - \theta_0)}\underline{l},$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\theta_0 + 2\frac{\theta_0}{(\beta M - \theta_0)} \times \underline{l}(x^*) - 4x^* = 0$, with the solution

$$x^*_{IIIa} = \frac{\theta_0(\beta M + \theta_0) - 2\beta(\theta_0 I - B_0)}{2(2\beta M - \theta_0)}. \tag{A2}$$

For (iii)(b), where $R > \bar{l} > \underline{l} > -R$, we have

$$E[\theta_1] = \frac{(\theta_0 - x)(R - \bar{l})}{4R} - \frac{(R^2 - \bar{l}^2)}{16R} + \frac{(\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - x))(\bar{l} - \underline{l})}{2R(\beta M - \theta_0)} + \frac{\theta_0(\bar{l}^2 - \underline{l}^2)}{4R(\beta M - \theta_0)},$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\theta_0 - 4x^* - \frac{(\theta_0 - x^*)(R - \bar{l}(x^*))}{2R} + \frac{(R^2 - \bar{l}^2(x^*))}{8R} - \frac{\theta_0(\bar{l}(x^*) - \underline{l}(x^*))^2}{2R(\beta M - \theta_0)} = 0$, with the solution

$$x^*_{IIIb} = \theta_0 + \frac{7R(3\theta_0 + \beta M)}{2(\beta M + 2\theta_0)} - \frac{\beta(\theta_0 I - B_0)}{(\beta M + 2\theta_0)} - \frac{\sqrt{\theta_0(M\beta + 3\theta_0)\big(4R\theta_0(6\theta_0(\beta M + 2\theta_0) - 7\beta(\theta_0 I - B_0)) + R^2\theta_0(48\beta M + 145\theta_0) + 4\beta^2(\theta_0 I - B_0)^2\big)}}{2\theta_0(\beta M + 2\theta_0)}. \tag{A3}$$

For (iv), where $\bar{l} > R > \underline{l} > -R$, we have

$$E[\theta_1] = \frac{\theta_0}{4R(\beta M - \theta_0)}(R - \underline{l})^2,$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\theta_0 - 4x^* - \frac{\theta_0}{2R(\beta M - \theta_0)}(R - \underline{l}(x^*))^2 = 0$, with the solution

$$x^*_{IV} = \theta_0 - \frac{R(4\beta M - 3\theta_0)}{\theta_0} - \frac{\beta(\theta_0 I - B_0)}{\theta_0} + \frac{\sqrt{2R(\beta M - \theta_0)\big(4R(2\beta M - \theta_0) - 3\theta_0^2 + 4\beta(\theta_0 I - B_0)\big)}}{\theta_0}. \tag{A4}$$

Solving $\bar{l}(x^*) = -R$ for B_0 using $x^* = 0$ gives $u_I \equiv \theta_0(I - \frac{M}{2} - \frac{3R}{4\beta} - \frac{\theta_0}{2\beta}) - \frac{RM}{4}$ for the cutoff debt level below which the arbitrageur is unconstrained. Solving $\underline{l}(x^*) = R$ for B_0 using $x^* = \frac{\theta_0}{4}$ gives $u_{IV} \equiv \theta_0(I - \frac{3\theta_0}{4\beta} + \frac{R}{\beta})$, the cutoff debt level above which the arbitrageur is always forced to liquidate.

When $\bar{l}(x^*) - \underline{l}(x^*) \geq 2R$, the boundaries are crossed in the order as follows: (i) $\bar{l} = -R$; (ii) $\bar{l} = R$; (iii) $\underline{l} = -R$; and (iv) $\underline{l} = R$. Solving $\bar{l} = R$ and $\underline{l}(x^*) = -R$ for B_0 using x^*_{IIIa} from (A2) gives the cutoff debt levels $u_{II} \equiv \theta_0 I - \frac{(\beta M + \theta_0)\theta_0}{2\beta} + \frac{(2\beta M - \theta_0)}{(\beta M - \theta_0)}\frac{(\beta M + 3\theta_0)R}{6\beta}$ and $u_{III} \equiv \theta_0 I - \frac{3(\beta M - \theta_0)\theta_0^2 + 2R\theta_0(2\beta M - \theta_0)}{4\beta(\beta M - \theta_0)}$. The condition $\bar{l}(x^*) - \underline{l}(x^*) \geq 2R$ must hold for all $u_{III} \geq B_0 > u_{II}$. Using x^*_{IIIa} in $\bar{l}(x^*)$ and $\underline{l}(x^*)$ gives

$$\frac{(4MI\beta^2 + 3M\beta\theta_0 - 3\theta_0^2)(\beta M - \theta_0)}{(\beta M + 3\theta_0)(2\beta M - \theta_0)} - \frac{4M\beta^2 B_0(\beta M - \theta_0)}{(\beta M + 3\theta_0)(2\beta M - \theta_0)\theta_0} \geq 4R.$$

The left-hand side decreases with B_0. Therefore, we must check that the condition holds at the highest value of B_0, which is u_{III}, giving the requirement that

$$\theta_0^2 - (\beta M - 4R)\theta_0 + \frac{2}{3}R\beta M \leq 0.$$

When $2R > \bar{l}(x^*) - \underline{l}(x^*) \geq 0$, the boundaries are crossed in the order as follows: (i) $\bar{l} = -R$; (ii) $\underline{l} = -R$; (iii) $\bar{l} = R$; and, (iv) $\underline{l} = R$. Solving $\underline{l}(x^*) = -R$ for B_0 using x^*_{II} from (A1) gives the cutoff debt level $u'_{II} = \theta_0 I + \frac{\theta_0 R}{2\beta}(\frac{5\beta M + 19\theta_0}{\beta M - \theta_0}) - \frac{\theta_0}{\beta}\frac{\sqrt{6R(2R(\beta M + 2\theta_0) + \theta_0(\beta M - \theta_0))(M\beta + 3\theta_0)}}{(\beta M - \theta_0)}$. Similarly, solving $\bar{l}(x^*) = R$ for B_0 using x^*_{IV} from (A4) yields the cutoff debt level $u'_{III} = \theta_0 I - \frac{\theta_0 R}{2\beta}(\frac{15\beta M + 17\theta_0}{\beta M - \theta_0}) + \frac{(\beta M + \theta_0)}{\beta} \times \frac{\sqrt{2R\theta_0(2R(\beta M + 15\theta_0) - 3\theta_0(\beta M - \theta_0))}}{(\beta M - \theta_0)}$. We obtain $\bar{l}(x^*) - \underline{l}(x^*) = \frac{(I\beta\theta_0 + \theta_0^2 - \beta B_0 - x^*\theta_0)(\beta M - \theta_0)}{(M\beta + 3\theta_0)\theta_0}$, which gives $\frac{\partial(\bar{l}(x^*) - \underline{l}(x^*))}{\partial B_0} = -\frac{(\beta M - \theta_0)}{(\beta M + 3\theta_0)\theta_0}(\beta + \theta_0\frac{\partial x^*}{\partial B_0}) < 0$ since $\frac{\partial x^*}{\partial B_0} = \frac{2\beta(\bar{l} - \underline{l})}{((7R + \bar{l})(\beta M - \theta_0) + 2\theta_0(\bar{l} - \underline{l}))} > 0$.

We need to check that $\bar{l}(x^*) - \underline{l}(x^*) < 2R$ at $\underline{l}(x^*) = -R$ with $B_0 = u'_{II}$, which produces the requirement that

$$\theta_0^2 - \theta_0(\beta M - 4R) + \frac{2}{3}R\beta M > 0.$$

We also need to check that $\bar{l}(x^*) - \underline{l}(x^*) \geq 0$ at $\bar{l}(x^*) = R$ with $B_0 = u'_{III}$; this holds if $\theta_0 > \frac{2}{3}R$.

If $R \geq (\frac{4 - \sqrt{7}}{12})\beta M$, then $2R \geq \bar{l} - \underline{l} > 0$ holds for all $\beta M > \theta_0 > \frac{2}{3}R$. For $(\frac{4 - \sqrt{7}}{12})\beta M > R, 2R \geq \bar{l} - \underline{l} > 0$ holds when $\beta M > \theta_0 > \frac{\beta M}{2} - 2R + \sqrt{(\frac{\beta M}{2})^2 - \frac{8}{3}R\beta M + 4R^2}$ and $\frac{\beta M}{2} - 2R + \sqrt{(\frac{\beta M}{2})^2 - \frac{8}{3}R\beta M + 4R^2} \geq \theta_0 \geq \frac{2}{3}R$; $\bar{l} - \underline{l} > 2R$ holds in case of $\frac{\beta M}{2} - 2R + \sqrt{(\frac{\beta M}{2})^2 - \frac{8}{3}R\beta M + 4R^2} \geq \theta_0 \geq \frac{\beta M}{2} - 2R - \sqrt{(\frac{\beta M}{2})^2 - \frac{8}{3}R\beta M + 4R^2}$. Q.E.D.

Proof of Corollary 1: We have $\bar{l} = 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)}$ and $\underline{l} = \frac{\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\theta_0}$. Partial differentiation with respect to B_0 yields $\frac{\partial \bar{l}(x^*)}{\partial B_0} = -\frac{2(\beta M + \theta_0)}{(\beta M + 3\theta_0)}\frac{\partial x^*}{\partial B_0} + \frac{4\beta}{(\beta M + 3\theta_0)}$ and $\frac{\partial \underline{l}(x^*)}{\partial B_0} = -\frac{\partial x^*}{\partial B_0} + \frac{\beta}{\theta_0}$.

(i) We have that x^*_{II} solves $\frac{(\beta M + 3\theta_0)}{8R(\beta M - \theta_0)}(R + \bar{l}(x^*))^2 - 3x^* = 0$. Differentiating with respect to B_0 gives $\frac{(\beta M + 3\theta_0)}{8R(\beta M - \theta_0)}(R + \bar{l}(x^*))\frac{\partial \bar{l}(x^*)}{\partial B_0} - 3\frac{\partial x^*}{\partial B_0} = 0$. Substituting in $\frac{\partial \bar{l}(x^*)}{\partial B_0}$ and rearranging terms, we get $\frac{\partial x^*}{\partial B_0} = \frac{2\beta(R + \bar{l}(x^*))}{((\beta M + \theta_0)(R + \bar{l}(x^*)) + 12R(\beta M - \theta_0))} \geq 0$.

(ii) (a) For $\theta_0^2 - \theta_0(\beta M - 4R) + \frac{2}{3}R\beta M \leq 0$, differentiating x^*_{IIIa} with respect to B_0 gives $\frac{\partial x^*}{\partial B_0} = \frac{\beta}{(2\beta M - \theta_0)} > 0$.

(ii) (b) For $\theta_0^2 - \theta_0(\beta M - 4R) + \frac{2}{3}R\beta M > 0$, we have $x^* = \frac{\theta_0}{4} - \frac{E[\theta_1]}{2}$ and $E[\theta_1] = \frac{1}{2R}\int_{\underline{l}}^{\bar{l}} \theta_1^C(\varepsilon)\,d\varepsilon + \frac{1}{2R}\int_{\bar{l}}^{R} \theta_1^U(\varepsilon)\,d\varepsilon$. This gives $\frac{\partial x^*}{\partial B_0} = \frac{2\beta(\bar{l} - \underline{l})}{((7R + \bar{l})(\beta M - \theta_0) + 2\theta_0(\bar{l} - \underline{l}))} > 0$.

(iii) We have that x^*_{IV} solves $\theta_0 - 4x^* - \frac{\theta_0}{2R(\beta M - \theta_0)}(R - \underline{l}(x^*))^2 = 0$. Differentiating with respect to B_0 gives $-4\frac{\partial x^*}{\partial B_0} + \frac{\theta_0(R - \underline{l}(x^*))}{R(\beta M - \theta_0)}\frac{\partial \underline{l}(x^*)}{\partial B_0} = 0$. Substituting in $\frac{\partial \underline{l}(x^*)}{\partial B_0}$ and rearranging terms, we get $\frac{\partial x^*}{\partial B_0} = \frac{\beta(R - \underline{l}(x^*))}{(4R(\beta M - \theta_0) + \theta_0(R - \underline{l}(x^*)))} \geq 0$. Q.E.D.

Proof of Corollary 2: Complete forced liquidation occurs when $\varepsilon_1 < \underline{l}$. When $\underline{l} \leq -R$, the probability of complete forced liquidation is 0, and when $\underline{l} \geq R$, the probability of complete forced liquidation is 1. For $R > \underline{l} > -R$, the probability of complete forced liquidation is $\frac{1}{2} + \frac{\underline{l}}{2R}$. Using $\underline{l} = \frac{\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\theta_0}$, we have that the probability of complete forced liquidation is $\frac{1}{2} + \frac{\theta_0}{2R} - \frac{\beta(\theta_0 I - B_0)}{2R\theta_0} - \frac{x}{2R}$. In the absence of the strategic trader, we have $x = 0$. In the presence of the strategic trader, we have from Proposition 1 that $x^* \geq 0$ when $\beta M > \theta_0 > 0$. This implies for any B_0, the probability of liquidation is lower in the presence of the strategic trader. This proves (i).

Over the range of values of ε_1 for which the constraint is binding, we have on rearranging terms, $\theta_1^* = \frac{\beta(\theta_0 I - B_0) - \theta_0(\theta_0 - \varepsilon_1)}{(\beta M - \theta_0)} + \frac{\theta_0 x^*}{(\beta M - \theta_0)}$. If the strategic trader is present, $x^* \geq 0$ when $\beta M > \theta_0 > 0$. Thus, the strategic trader's presence loosens the constraint, allowing the arbitrageur to liquidate a smaller amount. This proves (ii).

For values of B_0 such that the constraint is never binding, the presence or absence of the strategic trader makes no difference as the optimal strategic trader quantity is 0. For levels of B_0 such that the constraint is binding, we have previously in (ii) that the presence of the strategic trader reduces the amount that the arbitrageur needs to sell to meet the constraint. Selling by the arbitrageur places downward pressure on prices, so the reduced amount that needs to be sold results in higher prices. This proves (iii). Q.E.D.

Proof of Proposition 2: The proof follows the steps outlined in the proof of Proposition 1. We repeat (16) here for convenience

$$\theta_1 = \begin{cases} \frac{1}{2}\left(\theta_0 - x - \frac{1}{2}\varepsilon_1\right) & \text{if } \varepsilon_1 \geq \bar{L} \\ \frac{\theta_0(\theta_0 - x - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)} & \text{if } \bar{L} > \varepsilon_1 \geq \underline{L} \\ 0 & \text{if } \underline{L} > \varepsilon_1, \end{cases}$$

where $\bar{L} \equiv 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)}$ and $\underline{L} \equiv 2\frac{\beta M(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\beta M + \theta_0}$.

In computing $E[\theta_1]$ we need to consider five regions.

(i) Region I: $-R > \bar{L}$, where the arbitrageur is always unconstrained. In this case, $E[\theta_1] = \frac{1}{2}(\theta_0 - x)$ and $x_I^* = 0$.

(ii) Region II: $R > \bar{L} > -R > \underline{L}$, where the arbitrageur is sometimes constrained and otherwise unconstrained.
(iii) Region III: (a) $\bar{L} > R > -R > \underline{L}$, where the arbitrageur is always constrained; or, (b) $R > \bar{L} > \underline{L} > -R$, where the arbitrageur is unconstrained, constrained, or fully liquidates.
(iv) Region IV: $\bar{L} > R > \underline{L} > -R$, where the arbitrageur is sometimes constrained and otherwise fully liquidates.
(v) Region V: $\underline{L} > R$, where the arbitrageur always fully liquidates. In this case, $E[\theta_1] = 0$ and $x_V^* = \frac{\theta_0}{4}$.

For (ii), where $R > \bar{L} > -R > \underline{L}$, we have

$$E[\theta_1] = \frac{(R-\bar{L})}{4R}\left(\theta_0 - x - \frac{\bar{L}}{4} - \frac{R}{4}\right) + \frac{R+\bar{L}}{4R}\left((\theta_0 - x) + \frac{(\beta M + \theta_0)\bar{L} + 2\theta_0 R}{2(\theta_0 - \beta M)}\right),$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\frac{(3\theta_0 + \beta M)}{4(\theta_0 - \beta M)} \times \frac{(R + \bar{L}(x^*))^2}{2R} + 3x^* = 0$, which has the solution

$$x_{II}^* = \frac{(\beta M + \theta_0)\theta_0 - 2\beta(\theta_0 I - B_0)}{(\beta M + \theta_0)} + \frac{R}{2}\frac{(7\beta M - 5\theta_0)(\beta M + 3\theta_0)}{(\beta M + \theta_0)^2}$$
$$+ \frac{\sqrt{12R(\beta M - \theta_0)(\beta M + 3\theta_0)(2((\beta M + \theta_0)\theta_0 - 2\beta(\theta_0 I - B_0))(\beta M + \theta_0) + 2R(2\beta M - \theta_0)(\beta M + 3\theta_0))}}{2(\beta M + \theta_0)^2}. \tag{A5}$$

For (iii)(a), where $\bar{L} > R > -R > \underline{L}$, we have

$$E[\theta_1] = (\theta_0 - x) + \frac{(\theta_0 + \beta M)}{(\theta_0 - \beta M)}\frac{\underline{L}}{2},$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $-\theta_0 - \frac{(\theta_0 + \beta M)}{(\theta_0 - \beta M)} \times \underline{L}(x^*) - 2x^* = 0$, which has the solution

$$x_{IIIa}^* = \frac{\theta_0(\theta_0 + \beta M) - 2\beta(\theta_0 I - B_0)}{2(2\beta M - \theta_0)}. \tag{A6}$$

For (iii)(b), where $R > \bar{L} > \underline{L} > -R$, we have

$$E[\theta_1] = \frac{(\theta_0 - x)(R - \bar{L})}{4R} - \frac{(R^2 - \bar{L}^2)}{16R} + \frac{(\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0))(\bar{L} - \underline{L})}{2R(\theta_0 - \beta M)}$$
$$- \frac{\theta_0(\bar{L}^2 - \underline{L}^2)}{4R(\theta_0 - \beta M)},$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\theta_0 - \frac{(\theta_0 - x)(R - \bar{L})}{2R} + \frac{(R^2 - \bar{L}^2)}{8R} - \frac{(\theta_0(\theta_0 - x) - \beta(\theta_0 I - B_0))(\bar{L} - \underline{L})}{R(\theta_0 - \beta M)} + \frac{\theta_0(\bar{L}^2 - \underline{L}^2)}{2R(\theta_0 - \beta M)} - 4x^* = 0$, which has the solution

$$x^*_{IIIb} = \theta_0 + \frac{7R}{2} + \frac{4\beta\theta_0^2(\theta_0 I - B_0) - 14R\theta_0^3}{\left(\theta_0^3 - 7M\beta\theta_0^2 - 5M^2\beta^2\theta_0 - M^3\beta^3\right)}$$

$$+ \frac{(M\beta+\theta_0)\sqrt{(M\beta+3\theta_0)\begin{pmatrix} 4\theta_0\beta^2(I\theta_0 - B_0)^2 + 6\left(7M\beta\theta_0^2 - \theta_0^3 + M^3\beta^3 + 5M^2\beta^2\theta_0\right)\theta_0 R \\ -28(I\theta_0 - B_0)\beta\theta_0^2 R + \left(37\theta_0^3 + 84M\beta\theta_0^2 + 12M^3\beta^3 + 60M^2\beta^2\theta_0\right)R^2 \end{pmatrix}}}{\left(\theta_0^3 - 7M\beta\theta_0^2 - M^3\beta^3 - 5M^2\beta^2\theta_0\right)}. \tag{A7}$$

For (iv) where $\bar{L} > R > \underline{L} > -R$, we have

$$E[\theta_1] = \frac{(R - \underline{L})}{2R}\left(\theta_0 - x - \frac{R}{2}\right) - \frac{\beta M}{(\theta_0 - \beta M)}\frac{(R - \underline{L})^2}{4R},$$

which on substitution into $\theta_0 - 2E[\theta_1(x^*, \varepsilon_1)] - 4x^* = 0$ gives $\frac{\underline{L}}{R}\theta_0 - (\frac{3R+\underline{L}}{R})x^* + \frac{(R-\underline{L})}{2} + \frac{\beta M}{(\theta_0 - \beta M)}\frac{(R-\underline{L})^2}{2R} = 0$, which has the solution

$$x^*_{IV} = \theta_0 - \frac{R(4\beta M - 3\theta_0)(\beta M + \theta_0)^2}{4\theta_0^2\beta M} - \frac{2\beta\left(\theta_0^2 + \beta^2 M^2)(\theta_0 I - B_0\right)}{4\theta_0^2\beta M}$$

$$- \frac{\sqrt{(\beta M - \theta_0)(\beta M + \theta_0)^2\begin{pmatrix} R^2\left(16\beta^3M^3 + 24\theta_0\beta^2M^2 + \theta_0^2\beta M - 9\theta_0^3\right) + 4\beta^2(\beta M - \theta_0)(\theta_0 I - B_0)^2 \\ +4\beta R\left(\left(4\beta^2M^2 + \beta M\theta_0 + 3\theta_0^2\right)(\theta_0 I - B_0) - 6\theta_0^3 M\right)\end{pmatrix}}}{4\theta_0^2\beta M}. \tag{A8}$$

Solving $\bar{L} = -R$ for B_0 using $x^* = 0$ gives $U_I \equiv \theta_0(I - \frac{M}{2} - \frac{3R}{4\beta} - \frac{\theta_0}{2\beta}) - \frac{RM}{4}$, the cutoff debt level below which the arbitrageur is unconstrained. Solving $\underline{L} = R$ for B_0 using $x^* = \frac{\theta_0}{4}$ gives $U_{IV} \equiv \theta_0(I - \frac{3M}{4} + \frac{R}{2\beta}) + \frac{RM}{2}$, the cutoff debt level above which the arbitrageur always finds it optimal to liquidate.

When $\bar{L} - \underline{L} \geq 2R$, the boundaries are crossed in the order as follows: (i) $\bar{L} = -R$; (ii) $\bar{L} = R$; (iii) $\underline{L} = -R$; and, (iv) $\underline{L} = R$. Solving $\bar{L} = R$ and $\underline{L} = -R$ for B_0 using x^*_{IIIa} from (A6) gives cutoff debt levels $U_{II} \equiv \theta_0 I - \frac{(\beta M + \theta_0)\theta_0}{2\beta} + \frac{(\theta_0 - 2\beta M)}{(\theta_0 - \beta M)}\frac{(\beta M + 3\theta_0)R}{6\beta}$ and $U_{III} \equiv \theta_0 I - \frac{R}{2\beta}\frac{(\theta_0 - 2\beta M)(\theta_0 + \beta M)}{(\theta_0 - \beta M)} - \frac{3\theta_0\beta M}{2\beta}$. The condition $\bar{L} - \underline{L} \geq 2R$ must hold for all $U_{III} \geq B_0 \geq U_{II}$.

Using x^*_{IIIa} in $\bar{L}$ and $\underline{L}$ gives

$$\frac{4M\beta^2(\theta_0 - \beta M)}{(\theta_0 - 2\beta M)(\beta M + \theta_0)(M\beta + 3\theta_0)}B_0 - \frac{\theta_0\left(4MI\beta^2 + 3M\beta\theta_0 - 3\theta_0^2\right)(\theta_0 - \beta M)}{(\theta_0 - 2\beta M)(\beta M + \theta_0)(M\beta + 3\theta_0)} \geq 2R.$$

For $\theta_0 > 2\beta M$, the left-hand side increases with B_0, so we must check that the condition holds at the smallest value of B_0, which is U_{II}. For $2\beta M > \theta_0 > \beta M$, the left-hand side decreases with B_0, so we must check that the condition holds at the largest value of B_0, which is U_{III}. Both of these yield the requirement that $\theta_0^2 - \theta_0(2R + \beta M) - \frac{4}{3}MR\beta \geq 0$. Also, we must have that $U_{III} \geq U_{II}$, which requires that $(\theta_0^2 - \theta_0(2R + \beta M) - \frac{4}{3}MR\beta)(\theta_0 - 2M\beta) \geq 0$.

These conditions together give us the requirement

$$\theta_0 \geq \max\left(2M\beta, \frac{\beta M}{2} + R + \sqrt{\left(\frac{\beta M}{2}\right)^2 + \frac{7}{3}MR\beta + R^2}\right). \tag{A9}$$

Given R, β, and M such that $\frac{8}{3}R \geq \beta M$, we have $(R + \frac{\beta M}{2}) + \sqrt{R^2 + \frac{7}{3}R\beta M + (\frac{\beta M}{2})^2} \geq 2\beta M$, and (A9) reduces to $\theta_0 \geq \frac{\beta M}{2} + R + \sqrt{(\frac{\beta M}{2})^2 + \frac{7}{3}MR\beta + R^2}$, while for $\beta M > \frac{8}{3}R$ we must have $\theta_0 > 2\beta M$.

When $2R > \bar{L} - \underline{L} \geq 0$, the boundaries are crossed in the order as follows: (i) $\bar{L} = -R$; (ii) $\underline{L} = -R$; (iii) $\bar{L} = R$; and, (iv) $\underline{L} = R$. Solving $\underline{L} = -R$ for B_0 using x_{II}^* from (A5) gives the cutoff debt level, $U_{II}' \equiv \theta_0 I - \frac{R(\theta_0^2 + 17M\beta\theta_0 + 6M^2\beta^2)}{2\beta(\theta_0 - \beta M)} + \frac{\sqrt{6RM^2(M\beta + 3\theta_0)(2MR\beta + (4R + M\beta)\theta_0 - \theta_0^2)}}{(\theta_0 - \beta M)}$. Similarly, solving $\bar{L} = R$ for B_0 using x_{IV}^* from (A8) gives the cutoff debt level, $U_{III}' \equiv \theta_0 I - \frac{(2R(\theta_0^3 + 2\beta^3M^3) + 13MR\beta\theta_0(\theta_0 + \beta M))}{2(\theta_0 - M\beta)(M\beta + 2\theta_0)\beta} + \frac{(M\beta + \theta_0)^2\sqrt{2R(14MR\beta\theta_0 - 6\theta_0^3 + 12R\theta_0^2 + 3M\beta\theta_0^2 + 6M^2R\beta^2 + 3M^2\beta^2\theta_0)}}{2(M\beta + 2\theta_0)(\theta_0 - M\beta)\beta}$. Using $\bar{L} - \underline{L} = 2 \times \frac{(\theta_0 - M\beta)}{(M\beta + \theta_0)(M\beta + 3\theta_0)}(I\beta\theta_0 + \theta_0^2 - \beta B_0 - x\theta_0)$ we obtain $\frac{\partial(\bar{L} - \underline{L})}{\partial B_0} = -2\frac{(\theta_0 - M\beta)}{(M\beta - \theta_0)(M\beta + 3\theta_0)} \times (\beta + \theta_0\frac{\partial x}{\partial B_0}) < 0$ since $\frac{\partial x^*}{\partial B_0} = \frac{2\beta\theta_0(\bar{L} - \underline{L})}{((\theta_0^2 - \beta^2M^2)(\frac{7}{2}R - \frac{1}{2}\bar{L} + \underline{L}) + 2\theta_0\beta M(\bar{L} - \underline{L}))} > 0$. This requires $\bar{L} - \underline{L} < 2R$ at U_{II}', giving $(\theta_0^2 - (2R + \beta M)\theta_0 - \frac{4}{3}R\beta M) < 0$ and $\bar{L} - \underline{L} > 0$ at U_{III}', thereby giving $\theta_0 > \frac{2}{3}R$. Q.E.D.

Proof of Corollary 3: We have $\bar{L} = 2\frac{(\beta M + \theta_0)(\theta_0 - x) - 2\beta(\theta_0 I - B_0)}{(\beta M + 3\theta_0)}$, $\underline{L} = 2\frac{\beta M(\theta_0 - x) - \beta(\theta_0 I - B_0)}{\beta M + \theta_0}$, and $\theta_1^C = \frac{\theta_0(\theta_0 - x - \varepsilon_1) - \beta(\theta_0 I - B_0)}{(\theta_0 - \beta M)}$, which gives $\frac{\partial \bar{L}(x^*)}{\partial B_0} = -\frac{2(\beta M + \theta_0)}{(\beta M + 3\theta_0)}\frac{\partial x^*}{\partial B_0} + \frac{4\beta}{(\beta M + 3\theta_0)}$, $\frac{\partial \underline{L}(x^*)}{\partial B_0} = -\frac{2\beta M}{\beta M + \theta_0}\frac{\partial x^*}{\partial B_0} + \frac{2\beta}{\beta M + \theta_0}$, and $\frac{\partial \theta_1^C}{\partial B_0} = -\frac{\theta_0}{(\theta_0 - \beta M)}\frac{\partial x^*}{\partial B_0} + \frac{\beta}{(\theta_0 - \beta M)}$.

(i) We have that x_{II}^* solves $\frac{(3\theta_0 + \beta M)}{4(\theta_0 - \beta M)}\frac{(R + \bar{L}(x^*))^2}{2R} + 3x^* = 0$. Differentiating with respect to B_0 gives $\frac{\partial x^*}{\partial B_0} = -\frac{(3\theta_0 + \beta M)}{6(\theta_0 - \beta M)}\frac{(R + \bar{L}(x^*))}{2R}\frac{\partial \bar{L}(x^*)}{\partial B_0}$. In this region we must have $\frac{\partial \bar{L}(x^*)}{\partial B_0} > 0$; otherwise, the arbitrageur would become less constrained as its debt level rises. This implies that $\frac{\partial x^*}{\partial B_0} < 0$.

(ii) (a) We have that $x_{IIIa}^* = \frac{2\beta(\theta_0 I - B_0) - \theta_0(\theta_0 + \beta M)}{2(\theta_0 - 2\beta M)}$. Differentiating with respect to B_0 gives $\frac{\partial x^*}{\partial B_0} = -\frac{\beta}{(\theta_0 - 2\beta M)} < 0$.

(ii) (b) We have that $x^* = \frac{\theta_0}{4} - \frac{E[\theta_1]}{2}$ and $E[\theta_1] = \frac{1}{2R}\int_{\underline{L}}^{\bar{L}}\theta_1^C(\varepsilon)\,d\varepsilon + \frac{1}{2R}\int_{\bar{L}}^{R}\theta_1^U(\varepsilon)\,d\varepsilon$. Differentiating with respect to B_0 and simplifying gives $\frac{\partial x^*}{\partial B_0} = \frac{2\beta\theta_0(\bar{L} - \underline{L})}{((\theta_0^2 - \beta^2M^2)(\frac{7}{2}R - \frac{1}{2}\bar{L} + \underline{L}) + 2\theta_0\beta M(\bar{L} - \underline{L}))} > 0$; the numerator is greater than 0 because $\bar{L} \geq \underline{L}$ and the denominator is greater than 0 because $R \geq \bar{L} \geq \underline{L} \geq -R$.

(iii) The optimal x^*_{IV} solves $\frac{\underline{L}}{R}\theta_0 - (\frac{3R+\underline{L}}{R})x^* + \frac{(R-\underline{L})}{2} + \frac{\beta M}{(\theta_0-\beta M)}\frac{(R-\underline{L})^2}{2R} = 0$. Differentiating with respect to B_0 gives $(2\theta_0 - 2x^* + 2\frac{\beta M}{(\theta_0-\beta M)}\underline{L} - (\frac{\theta_0+\beta M}{\theta_0-\beta M})R)\frac{\partial \underline{L}(x^*)}{\partial B_0} - (6R + 2L_{L2})\frac{\partial x^*}{\partial B_0} = 0$. Substituting in $\frac{\partial \underline{L}(x^*)}{\partial B_0}$ and rearranging terms, we get $\frac{\partial x^*}{\partial B_0} = (\frac{(6R+2\underline{L})(\theta_0-\beta M)(\theta_0+\beta M)}{2\beta(2(\theta_0-x^*)(\theta_0-\beta M)+2\beta M\underline{L}-(\theta_0+\beta M)R)} + M)^{-1} > 0$ using the fact that $\check{L}(x^*) \geq -R$, $\theta_0 > 2\beta M$ and $\theta_0 > x^*$.

For $\bar{L} - \underline{L} \geq 2R$, we have $x^*_{IV} = -\frac{\theta_0}{2} + \frac{R(\theta_0+\beta M)}{2(\theta_0-\beta M)} < 0$ at $B_0 = U_{III}$ and $x^*_{IV} = \frac{\theta_0}{4} > 0$ at $B_0 = U_{IV}$. Continuity of x^*_{IV} and the fact that $\frac{\partial x^*}{\partial B_0} > 0$ gives us that $x^*_{IV} = 0$ for at least one $B_0 \equiv B_0(0) \in (U_{III}, U_{IV})$. Setting $x^* = 0$ in (A8), we have that $B_0(0)$ solves

$$2\theta_0\underline{L}(0)(\theta_0 - \beta M) + (R\theta_0 - \underline{L}(0))(R - \beta M\underline{L}(0)) = 0. \tag{A10}$$

For $\bar{L} - \underline{L} < 2R$, we have $x^*_{III} \leq 0$ at $B_0 = U_{II}$ and $x^*_{IV} = \frac{\theta_0}{4} > 0$ at $B_0 = U_{IV}$. Continuity of x^* and the fact that $\frac{\partial x^*}{\partial B_0} > 0$ gives us $x^* = 0$ for at least one $B_0 \equiv B_0(0) \in (U'_{II}, U_{IV})$. If $x^*_{III} < 0$ at $B_0 = U'_{III}$, then $B_0(0) > \theta_0 I + \frac{(\beta M+3\theta_0)R - 2(\beta M+\theta_0)\theta_0}{4\beta}$, and $B_0(0)$ solves (A10). Otherwise, $B_0(0)$ solves

$$\begin{aligned} &R(4\theta_0 + R)(\theta_0 - \beta M) - (\beta M + 3\theta_0)\bar{L}^2(0) \\ &\quad + 4\underline{L}(0)\big(2\theta_0^2 - 2\beta(\theta_0 I - B_0) - \theta_0\underline{L}(0)\big) = 0. \end{aligned} \tag{A11}$$

Q.E.D.

Proof of Corollary 4: The probability of full liquidation is $\frac{\underline{L}+R}{2R}$. In the absence of the strategic trader, this is $\frac{1}{2} + \frac{\beta M\theta_0 - \beta(\theta_0 I - B_0)}{(\beta M+\theta_0)R}$. In the presence of the strategic trader, the probability is $\frac{1}{2} + \frac{\beta M\theta_0 - \beta(\theta_0 I - B_0)}{(\beta M+\theta_0)R} - \frac{\beta M}{(\beta M+\theta_0)R}x$. When $x < 0$, the presence of the strategic trader causes an increased probability of full liquidation; whereas, for $x > 0$, the presence of the strategic trader causes a reduced probability of full liquidation. Thus, for $B_0 < B(0)$, the presence of the strategic trader causes an increase in the probability of full liquidation, whereas for $B_0 > B(0)$ the presence of the strategic trader causes a reduction in the probability of full liquidation. This proves (i).

Part (ii) follows directly from (i). Q.E.D.

Proof of Corollary 5:

(a) To show the result, we compare the maximum and minimum possible prices at $t = 1$ with and without strategic trading.

The maximum possible price is attained at the lowest value for ε_1 that avoids full liquidation by the arbitrageur. The price $P_1^{\max}$ is then $P_1^{\max} = I + \frac{\theta_0\beta M - \beta(\theta_0 I - B_0)}{\beta(\theta_0-\beta M)} - \frac{\beta M}{\beta(\theta_0-\beta M)}(2\frac{\beta M\theta_0 - \beta(\theta_0 I - B_0)}{\beta M+\theta_0} - \frac{2\beta M x^*_{IV}}{\beta M+\theta_0}) - \frac{\beta M x^*_{IV}}{\beta(\theta_0-\beta M)}$. The difference between this expression assuming a nonzero x^*_{IV} (with strategic trading) and this expression with $x^*_{IV} = 0$ (without strategic trading) is $\frac{\beta M}{\beta(\theta_0-\beta M)}x^*_{IV}(\frac{2\beta M}{\beta M+\theta_0} - 1)$. Given that $\theta_0 > \beta M$ for a large position, $\frac{\beta M}{\beta(\theta_0 - \beta M)} > 0$ and $\frac{2\beta M}{\beta M + \theta_0} - 1 < 0$. Therefore, the expression has the opposite sign of x_{IV}^*, which implies that the highest possible price is higher (lower) in the presence of strategic trading if x_{IV}^* is negative (positive).

The minimum possible price prevails at $\varepsilon_1 = -R$. The difference between the minimum price in the presence and the absence of strategic trading is $\frac{x_{IV}^*}{\beta}$. This implies that the lowest possible price is lower (higher) in the presence of strategic trading if x_{IV}^* is negative (positive).

(b) We first prove the claim for $t = 1$ and $x_{IV}^* < 0$. Suppose first that the arbitrageur trades to meet the constraint. In a market without strategic trading, the price must be higher when the arbitrageur is constrained than when it is unconstrained. This follows immediately from a constrained arbitrageur's higher holdings to meet the capital requirement. In this regime, the difference in prices between a market with strategic trading and without for a given ε_1 is $-\frac{\beta M x_{IV}^*}{\beta(\theta_0 - \beta M)}$, which is strictly positive for $x_{IV}^* < 0$. This proves that in this regime strategic trading distorts prices further upward. Suppose now that the arbitrageur collapses. Given that the price at $t = 1$ when the arbitrageur is unconstrained can be written as $I - \frac{1}{2}(\frac{\theta_0}{\beta} + \frac{3}{2}\frac{\varepsilon_1}{\beta})$, the difference between the price when the arbitrageur is unconstrained and when it is constrained computes as $\frac{1}{2}(\frac{\theta_0 - \frac{1}{2}\varepsilon_1}{\beta} - 2\frac{x_{IV}^*}{\beta})$. It is straightforward to see that the expression is decreasing in x_{IV}^*, and, using the assumption $\theta_0 > \frac{1}{2}R$, that it is positive for $x_{IV}^* < 0$. This shows that in this regime strategic trading distorts prices further downward.

The proof for $t = 1$ and $x_{IV}^* > 0$ can be established in a similar way and for brevity is omitted here.

Note that the aggregate trading across the two dates by the arbitrageur and the strategic trader is constant, $-\theta_0$. This implies that strategic trading distorts prices in $t = 2$ in exactly the same magnitude as in $t = 1$ but in the opposite direction. This proves the claim for $t = 2$. Q.E.D.

Proof of Proposition 3: Since a loan does not carry interest, the objective function of the arbitrageur when obtaining a loan from the strategic trader continues to be given by (10). The initial debt level, B_0, enters the objective function as a constant. Thus, B_0 affects the arbitrageur's decision only through the constraint imposed by the capital requirement. When $b_1 \geq 0$ denotes the amount lent by the strategic trader at time 1, the constraint is modified to

$$\theta_0\left(I + \frac{x_1 + \varepsilon_1}{\beta}\right) - (B_0 - b_1) \geq \theta_1 M - \frac{\theta_0}{\beta}(\theta_1 - \theta_0). \tag{A12}$$

This expression is identical to the constraint without a loan from the strategic trader if the initial debt were $B_0 - b_1$. This implies that the arbitrageur's trading behavior as well as the strategic trader's trading profits with lending are identical to a situation without lending but with a reduced debt amount.

The strategic trader lends if and only if doing so increases trading profits. Solving (9) for $E(\theta_1)$ and inserting the result in (8) reveals that its trading profits are given by $2\frac{x^{*2}}{\beta}$ and are therefore positively related to the amount of its trading. From Corollary 1 we know that for an arbitrageur with a small position, the strategic trader's volume of trading and thus its expected trading profit never increases as the arbitrageur's initial debt declines. Therefore, if the arbitrageur holds a small position, lending does not increase trading profits and does not occur in equilibrium. If the arbitrageur holds a large position, however, the strategic trader's trading volume and profit as a function of B_0 are nonmonotonic (Corollary 3). Thus, there must be levels of B_0 for which lending increases trading profits.

To show that situations exist in which both the strategic trader and the arbitrageur are better off if a loan is made, consider a debt level $B_0 > U_{III}$ of the arbitrageur that, without lending by the strategic trader, leads to $x^* \leq 0$. Denote this level of trading by the strategic trader by x°. Given the shape of x^* as a function of B_0, there exists a smaller B_0, B_0°, that leads also to x°, implying identical expected profits for the strategic trader. Suppose that the size of a loan by the strategic trader is designed to move the arbitrageur to where it trades as though it has an initial debt level of B_0°; denote this loan amount by b_1°. Given that the strategic trader's behavior is unchanged, the loan increases the arbitrageur's expected profit by a discrete amount. The continuity of the arbitrageur's expected profit function implies that there is a loan amount $b_1^\circ - \eta$ with $\eta > 0$ such that the strategic trader's profits increase strictly when lending and the arbitrageur accepts the loan. Thus, lending takes place in equilibrium. Q.E.D.

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