Time to Wind Down: Closing Decisions and High Water Marks in Hedge Fund Management Contracts*

Martin Ruckes Margarita Sevostiyanova

Karlsruhe Institute of Technology

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Abstract

This paper provides a rationale for the inclusion of high water mark provisions in hedge fund management contracts. When hedge fund managers are better informed about future fund profitability than investors, contracts including high water marks provide the fund managers with better incentives to efficiently close the fund than contracts with linear performance fees only. The model implies that funds with high water marks tend to close more quickly upon periods of poor performance than their period performance fee counterparts. If, however, such funds with high water mark arrangement decide to continue, their after-fee performance is expected to be superior to comparable funds employing period performance fees. The model is also consistent with empirical evidence that high water marks are more common in smaller funds and funds run by managers without extensive track records.

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1 Introduction

The literature on dynamic incentive provision typically proposes contracts that specify compensation that is based on outcomes during the time period the manager can affect these outcomes. For example, it is argued that a manager should be compensated for a period's activities exclusively based on that period's outcomes rather than on previous periods' results as this may distort current incentives (see, for example, Holmström and Milgrom, 1987). Hedge fund management contracts typically violate this property in that the performance component of the management contract specifies that a performance fee is based on the fund value at the end of a given period relative to the fund value's historic maximum rather than that at the beginning of the period.[1] This fee structure is frequently referred to as a performance fee with a high water mark provision. In addition to performance fees, most hedge fund management contracts also specify a so called management fee that is based on the value of assets under management.

The historic fund value and the fund value at the beginning of a period differ only when periods of losses have occurred in the past. Then, a high water mark provision has two effects compared to an otherwise identically structured fee based on period performance: 1) it reduces the expected fee amount paid to the hedge fund manager as a fee is applied to a smaller base and 2) it introduces a convexity in the fee structure as a fee is only paid on period performance above a strictly positive level. In a seminal contribution, Panageas and Westerfield (2009) argue that these properties of a high water mark provision may have desirable incentive effects in a dynamic context. They show that under a high water mark contract, a risk neutral hedge fund manager displays risk averse behavior provided that the fund's horizon is sufficiently long. In case of a long fund horizon, a large share of the fund manager's expected income stems from future fees. As future fees are reduced when the fund's value is below its historic maximum, the manager tries to avoid reaching such states. He will do so by limiting the risk of the fund's holdings.

When the fund's horizon is short, however, a high water mark provision's implications for managerial risk taking are much less clear. Specifically, a high water mark may lead to excessive risk taking by the hedge fund manager due to the convexity of the fee structure (Hodder and Jackwerth, 2007, and Chakraborty and Ray, 2008). Indeed, the average life span of a hedge fund is rather short. According to Malkiel and Saha (2005), annual hedge fund attrition rates in the years 1994 to 2003 have been below 10 percent only in one of

[1] In the comprehensive sample of Agarwal et al. (2009), 80.1 percent of hedge funds display such a fee structure.

the years.[2]

In this paper we argue that high water mark provisions have desirable incentive effects especially for hedge funds with limited but uncertain horizons. They do so because high water mark provisions facilitate the efficient closing of hedge funds by their managers.[3]

The profitability of a hedge fund's strategy changes over time. At any point in time the hedge fund's manager is typically in a better position than investors to identify whether the prospects of the fund's strategy warrant the fund's continuation. For example, while investors may have to infer the quality of a fund's strategy from recent performance its manager possesses in depth knowledge of the fund's strategy and holdings. Combined with a close following of the markets relevant to the strategy this typically allows her to better assess whether the recent performance tends to be temporary in nature or indicates a permanent change of fund prospects.

A fund manager's incentives to close the fund are not necessarily aligned with those of investors. Especially because negative fee payments are normally impossible to enforce, even funds with poor prospects may generate significantly positive expected fees for the manager. Thus, a management contract that leads to efficient fund closing needs to specify low expected fees in circumstances in which fund closure may be efficient. Since this is typically the case when recent performance has been poor, the high water mark's effect of reducing expected fees in these situations improves managerial incentives to close the fund. Anticipating a more efficient fund closing decision increases investors' willingness to provide capital to the fund and in turn tends to increase expected fees for the fund manager. The property that a management contract with high water mark generates relatively low expected fees when fund performance has been poor, allows the manager to set a relatively high performance fee rate. Doing so mitigates a second type of incentive problem. Because the manager does not fully participate in the value gains of a fund, she may close it even when the fund has performed well and fund prospects are intact. A high performance fee rate implies high expected future fees when the fund value is at its historic maximum, which leads to a low probability of fund closing.

We present a model that formally characterizes the above argument and show that linear

[2] Chan et al. (2005) report that for their sample of hedge fund liquidations "... half of all liquidated funds never reached their fourth anniversary." Note that fund liquidation does not necessarily mean failure; see Liang and Park (2010).

[3] The notion that fund closings are frequently instigated by fund management, is reflected, for example, in the closing announcement of Atticus Global Fund: "This decision will come as a surprise to most of you, especially given that we have received redemptions of less than 5% of capital ... ."

performance fee contracts with high water marks dominate those with period performance fees when fee levels are set endogenously. Our approach implies that funds with high water marks tend to close with a higher probability upon periods of poor performance than their period performance fee counterparts. If, however, such funds with high water mark arrangements decide to continue, their after-fee performance levels are expected to be superior to comparable funds employing period performance fees.

In our model, performance fee levels are set to optimize managerial incentives whereas management fees are typically used by the fund manager to extract rents. Optimal management fees are lower if the performance fee structure contains a high water mark than if it does not. In the former case, the non-negativity constraint of the management fee may even be binding. Then, the performance fee serves also as the instrument for manager to extract rents. When the probability of a deterioration of the fund's prospects upon poor performance is sufficiently low, a contract with period performance fee is even preferred over that with high water mark. Given that small funds and funds run by managers that lack extensive track records can be associated with relatively high probabilities of deterioration of prospects such funds are expected to more frequently employ performance fees with high water mark provisions.

Related Literature

Aragon and Qian (2010) provide a rationale for the inclusion of high water mark provisions in hedge fund management contracts based on *ex ante* asymmetric information. Hedge fund managers attempt to credibly signal their quality by offering a contract that pays lower expected fees when performance is poor. As a contract containing a high water mark tends to imply a lower fee for several periods, it is particularly well suited to be used as a signaling device. Aragon and Qian (2010) show that high water marks can reduce excessive closing caused by investor redemptions. In contrast, our approach focuses on the closing decision by fund management and argues that high water marks not only reduces excessive continuation by fund managers upon poor performance but also excessive termination after strong returns.[4] Also, in the model presented here an informational asymmetry arises after the contract is signed rather than beforehand.

In our approach, the use of high water mark provisions plays a significant role in hedge funds' closing decisions. Empirical studies confirm the impact of high water marks on

[4]While the latter benefit of high water marks may appear of less economic consequence, cases of fund closings by managers after strong results do exist. For example, Andrew Lahde, founder of Lahde Capital, decided to close down his funds and to return money to investors after a return of 870 percent the previous year ["Hedge fund returns money", Financial Times Online, September 22nd, 2008].

closure rates of funds. Brown, Goetzmann and Park (2001), Aragon and Nanda (2009) as well as Ray (2009) document that hedge funds whose value is further below their high water marks close at higher rates. Anecdotal evidence also points towards the influence of future expected fees on closing decisions: "Most funds close down because it does not pay their managers to continue, not because their performance has been disastrous."[5] Particularly the high water mark contract component is thought to be responsible for this behavior: "[The fact that they are still below their peak performance] has lead many hedge funds to wind down rather than attempt to claw their way back to the point at which they can earn performance fees."[6] Liang and Park (2010) find that hedge funds with high water marks tend to close more quickly upon bad performance.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 is concerned with the optimal contract design when performance fee levels are set to optimize managerial incentives and management fees are used by the fund manager to extract rents. In Section 4, additional empirical predictions related to fund closing, after-fee fund performance and the level of management fees are derived. Section 5 studies the parameter set in which the manager employs the performance fee also to extract rents. In Section 6 we provide some robustness analysis by allowing for intermittent capital redemptions and capital contributions by the fund manager. Section 7 concludes.

2 A Simple Model

We describe a stylized two-period model of hedge fund management contracting. During the first period, information about the quality of the fund's strategy is revealed which may lead to a subsequent closing of the fund.

Fund Manager and Investor

Consider an investment manager who has an idea for an investment strategy with a time horizon of two periods. The investment strategy is limited in scale: cash returns are linear in initial investment, but any initial amount above $V_0 = 1$ cannot be invested profitably. The manager does not have financial wealth of her own and needs to raise capital from an investor to implement her investment strategy. There exists an outside investor who has one unit of capital to invest. Alternatively to operating a hedge fund in the second period,

[5] "Hedge podge", Economist, February 16th, 2008.

[6] "Atticus closes flagship fund", Financial Times Online, August 11th, 2009.

the manager has a valuable opportunity to obtain an outside income of ω from working in a different occupation, if the fund is inactive. This outside income ω is privately observed by the manager at the beginning of the second period. Ex ante, ω is distributed according to a cumulative distribution function $F(\omega)$ with density $f(\omega)$ on a support of $[0, \omega_{max}]$.

The manager is assumed to have the entire bargaining power vis a vis the investor and thus to make a take-it-or-leave-it offer to the investor. Both parties are risk neutral and the risk free interest rate is zero.

Characteristics of Investment Strategy and Beliefs

Implementing the manager's strategy implies that in each period, the invested amount yields either a positive return $R^H > 0$ or a negative return $R^L < 0$. Consequently, fund values after the first period are either $V^H \equiv 1 + R^H$ or $V^L \equiv 1 + R^L$ and after the second period $V^{HH} \equiv (1 + R^H)^2$, $V^{LL} \equiv (1 + R^L)^2$, or $V^{LH} \equiv (1 + R^L)(1 + R^H) \equiv V^{HL}$, where the first and second superscripts indicate the realized return in the first and second periods, respectively. Returns are costlessly verifiable. At the time of contracting at date 0 the investor and the manager are symmetrically informed about the probabilities for both positive and negative returns of the fund in the first period $p \in (0, 1)$ and $(1 - p)$, respectively. It holds that

$$\eta := pR^H + (1 - p)R^L > 0, \qquad \text{(A1)}$$

i.e. the investment strategy is profitable in the first period.

Investing in the first period generates information about expected second-period returns. While a positive first-period return (state H) is uninformative for second-period prospects, a negative first-period return tends to be associated with a deterioration of expected returns.[7] Concretely, given a negative first-period return, one of two return distributions may materialize: one where the first-period return is a purely temporary phenomenon and the probability of a positive second-period return remains at p (state L°), and one where the first-period return is indicative for second-period prospects and the probability of a positive second-period return decreases to $p - \varepsilon$ (state L^{-}), with $\varepsilon \in (0, p)$. As we are interested in whether fund closing takes place efficiently, we assume that the lowest probability of a positive return in the second-period, $p - \varepsilon$ implies a negative expected surplus:

$$(p - \varepsilon)R^H + (1 - p + \varepsilon)R^L < 0. \qquad \text{(A2)}$$

[7] Assuming that a positive first-period return tends to be associated with an improvement of prospects does not affect the results.

Assumption (A2) implies that it is efficient to close the fund if the investment strategy's prospects have deteriorated irrespective of the manager's realisation of outside option. A change in prospects occurs with probability $1-\theta$ in case of a negative first-period return (see Figure 1).

We also assume that it is efficient to continue the fund after a positive first-period return:

$$\left(1+R^{H}\right)\eta \geq \omega_{max}. \qquad \text{(A3)}$$

While assumption (A3) appears natural, it illustrates a central economic notion underlying the model. When the fund has been doing well, investors want to see the fund continued. In that case, the only possibly relevant distorting behavior by the manager is to close the fund with positive probability.

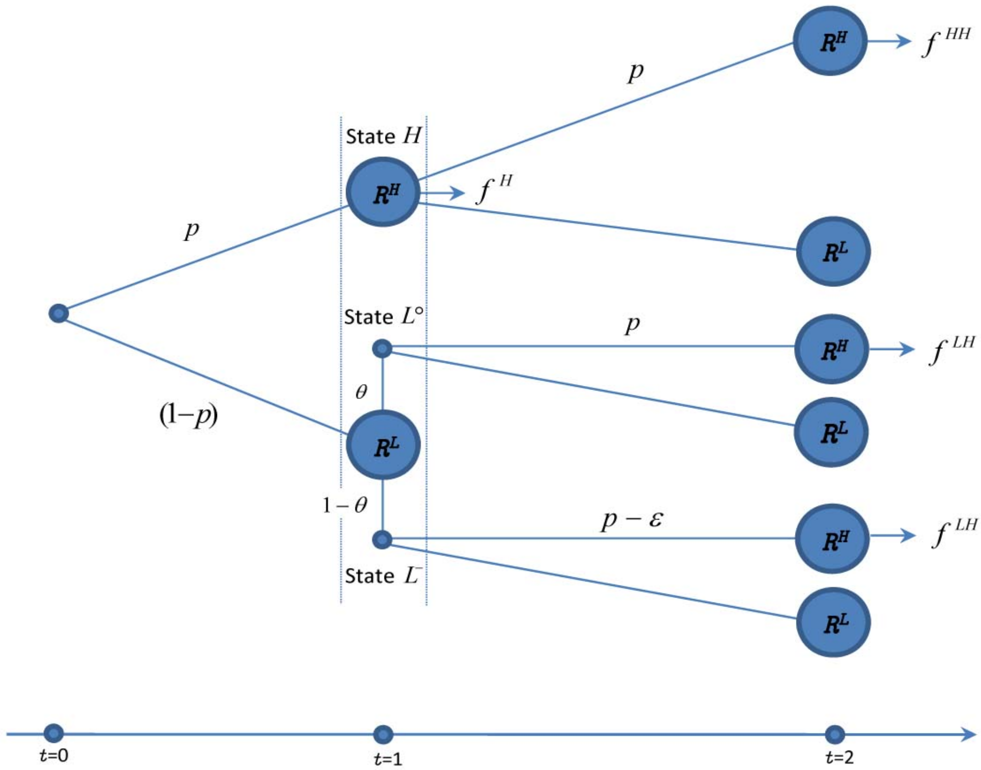

Figure 1: Return probabilities.
Ex ante view of states H, $L°$ and L^-, success probabilities in the first and second periods and corresponding fees in all outcomes after positive period return R^H.

Due to the intimate knowledge of her own investment strategy and close observation of market development, the manager observes the true return distribution arising at date 1. The investor is unable to observe the true return distribution and is only informed about the first-period return of the fund. Upon observing a positive first-period return, the investor's probability for a positive second-period return remains at p and the probability of a positive second-period return is $p - \varepsilon(1-\theta)$ when he observes a negative first-period return.

Compensation and Fund Closing

While ω characterizes the manager's income during the second period if the fund is inactive,[8] the manager is compensated in the form of ex ante specified fees during active periods. The fees can be made contingent on the fund's performance. Due to the manager's limited wealth, fees cannot be negative in any period. This implies that after a fee has been paid out, they are unaccessible to the investor in later periods.

We focus on two performance-based compensation arrangements:

- a *period performance fee*, where at the end of each period the manager receives a constant fraction of the fund's value gain during the period and nothing when the fund loses value during the period,
- a *performance fee with a high water mark provision*, where at the end of each period the manager receives a constant fraction of the fund's value gain during the period relative to the fund's historic maximum value and nothing when the fund's value is below its historic maximum.

In addition to the performance fee, the manager charges a one-time management fee $k \geq 0$.[9] In the following, we assume the non-negativity constraint to be not binding in the optimal contract. Allowing for a fixed management fee in that way enables us to separate the incentive effects of performance fees from the expected level of compensation. We examine the case in which the inequality is binding in Section 5.

As either of the two performance-sensitive arrangements specifies a payment of zero to the manager in case of a negative return during the period, at maximum three payments have to be specified. The contractually agreed performance-based payment upon a positive

[8]Introducing a positive outside income in the first period, does not affect the results as long as the fund is still launched.

[9]In practice, management fees are frequently paid periodically as a fraction of the assets under management. One purpose of the management fee is to cover a fund's operational expenses. For example, the investors in the funds Citadel Kensington Global Strategies and Citadel Wellington bear all the funds' expenses directly in place of paying a management fee [see "Citadel Discusses Fees, Redemptions," *Wall Street Journal Online*, September 10th, 2010]. Findings by Deuskar et al. (2011), however, indicate that a fund's improvement in perceived quality tends to allow it to increase its management fee.

first-period return is denoted by f^H. Second-period performance-based payments are dependent on first-period returns and are denoted by f^{HH} and f^{LH}, with the first and second superscripts denoting the first-period and second-period returns, respectively (see Figure 1). To simplify the analysis, it is assumed that the investor pays any fees separately from the fund to the manager.[10]

Due to the inalienability of human capital, the manager cannot be forced to continue the fund after period 1. Thus, the manager can close the fund at that date. It is not possible to specify a fee that is contingent on the manager's decision to close the fund. Alternatively to the manager, the investor is able to effectively close the fund at date 1 if he is allowed to withdraw her capital from the fund. While hedge funds typically allow investors to withdraw capital, many funds impose material restrictions on redemptions. For example, "lock-up provisions" specify the time period that an investor has to at least leave his capital in the fund for and "gates" limit the amount of funds that can be withdrawn within a certain time span at the investor and/or the fund level.[11] In the following, we analyze a situation in which the investor is not permitted to withdraw capital from the fund. In Section 6, we discuss if it can be optimal to allow the investor to close the fund by redeeming his capital.

3 The Fund Management Contract

The sequence of events is as follows (see Figure 2). At date 0 the contract is signed and investors provide financial capital. The fund manager invests this capital according to her identified strategy. At date 1 the first-period return is observed by all parties and the fee to the manager is paid as specified in the fund management contract. Then the manager learns about her expected outside income ω and decides whether to continue the fund or close it. If the fund is closed, all assets are liquidated at no cost and the proceeds are paid to the investor. If the fund remains alive, assets are used according to the investment strategy. At date 2, an alive fund's return is observed, its assets are costlessly liquidated and the proceeds distributed to the investor. The contractually agreed fee is paid to the

[10] As long as the fund's assets are sufficiently liquid, assuming that the cash to pay the fees are generated by liquidating the corresponding part of the fund's assets does not change the results.

[11] Ang and Bollen (2009) compute that the cost of lockup provisions and withdrawal suspensions can be significant for investors. Lock-up provisions and gates vary significantly with the liquidity of investments. For example, in FrontPoint Partners' FrontPoint-SJC Direct Lending Fund investors' funds are locked up for five years. ["FrontPoint raises $1bn for new fund", Financial Times Online, January 7, 2011.]

manager. If the fund is closed at date 1, the manager receives her outside income ω at date 2.

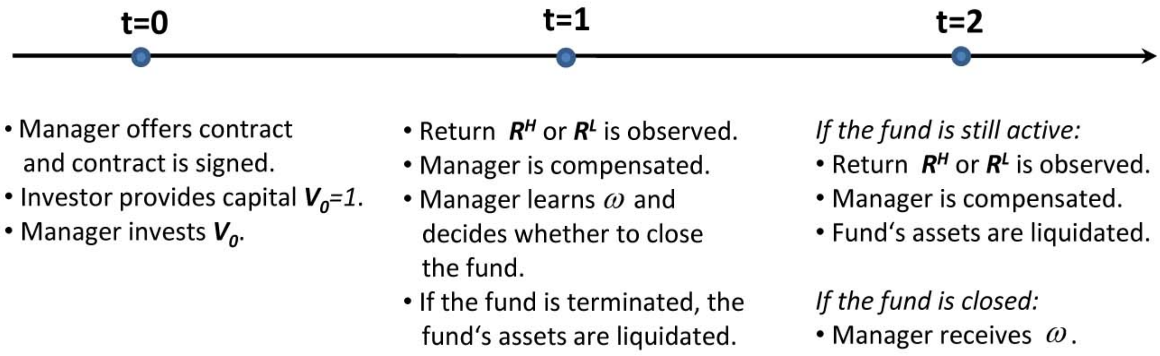

Figure 2: Sequence of events .

Analysis

We first describe the fund manager's optimization problem independent of the specific structures of the performance fee discussed above. In this analysis, we represent the performance fee structure in the fund management contract by $\mathcal{A}$. Subsequently, we compare the outcomes when using a period performance fee and a performance fee with high water mark.

As the participation constraint of the investor can be satisfied by adjusting the fixed performance fee, k, the performance fee structure serves two potential conflicts of interest between investor and manager with respect to closing the fund. There is a potential incentive for the manager to continue the fund even though doing so is not in the interest of the investor, because she does not explicitly participate in losses the fund suffers. The only way the performance fee arrangement is able to control this incentive is by specifying relatively low expected future fees in the relevant states. The incentive for excessive continuation is present in state L^- and possibly in state L°, because negative first-period returns tend to be the consequence of a worsening of fund prospects. There is also a potential incentive to close the fund even though the investor would like to see it continued. This is, because the manager participates only with a certain fraction in the expected value gains of the fund. The contract can mitigate this incentive by offering relatively high expected fees in the relevant states. The incentive for excessive closing is present in state H and possibly in state L°.

To identify the optimal fund management contract, we first describe the manager's closing decision at date 1 and the investor's participation constraint as well as the manager's objective function in general.

The manager decides whether or not to close the fund at date 1 based on the realization of her outside income ω. She closes the fund whenever her outside income in period 2 equals or exceeds her expected second-period fee income from operating the fund. For any given fee structure and each of the three states at date 1, H, L° and L^{-}, there is a level of ω above which the manager closes the fund. Those cutoff levels depend on the performance fee arrangement are denoted by $\omega^{H}(\mathcal{A})$, $\omega^{L^{\circ}}(\mathcal{A})$ and $\omega^{L^{-}}(\mathcal{A})$. Given our possible fee structures, it holds $\omega^{H}(\mathcal{A}) \geq \omega^{L^{\circ}}(\mathcal{A}) \geq \omega^{L^{-}}(\mathcal{A})$. Then, the manager's closing decision can be characterized as follows: At date 1, the manager

never closes	for	$\omega < \omega^{L^{-}}(\mathcal{A})$
closes iff $prob(R^{H}) = p - \varepsilon$	for	$\omega \geq \omega^{L^{-}}(\mathcal{A})$
closes iff first period return is R^{L} and $prob(R^{H}) = p$	for	$\omega \geq \omega^{L^{\circ}}(\mathcal{A})$
always closes	for	$\omega \geq \omega^{H}(\mathcal{A})$.

The investor's participation constraint depends on his anticipation of the manager's closing decision. The investor's participation constraint depends on the fund's performance and both the management fee, k, as well as the performance fee arrangement. The performance fee arrangement affects the investors payoff not only through fee payments to the manager but also via the manager's closing choice at date 1. We drop the descriptor $(\mathcal{A})$ for brevity. The investor's participation constraint is then given by

$$\begin{aligned}
&V_0 \leq -k + \\
&+p\Big(-f^{H}+F(\omega^{H})\big(p(V^{HH}-f^{HH})+(1-p)V^{HL}\big)+(1-F(\omega^{H}))V^{H}\Big) + \\
&+(1-p)\theta\Big(F(\omega^{L^{\circ}})\left(p(V^{LH}-f^{LH})+(1-p)V^{LL}\right)+(1-F(\omega^{L^{\circ}}))V^{L}\Big) + \\
&+(1-p)(1-\theta)\Big(F(\omega^{L^{-}})\big((p-\varepsilon)(V^{LH}-f^{LH})+(1-p+\varepsilon)V^{LL}\big)+(1-F(\omega^{L^{-}}))V^{L}\Big).
\end{aligned} \tag{1}$$

While the first line of (1) contains the fixed management fee to be paid to the manager, lines 2 to 4 describe investor's payoffs in the three states weighted with the probabilities with which the states occur. Each payoff depends on the fee structure both directly and indirectly via the fee structure's impact on the manager's closing decision. In equilibrium, the manager will set the management fee, k, to its maximum value provided that the investor is willing to provide capital. Therefore, the investor just breaks even in equilibrium and (1) is fulfilled with equality.

Because the fund manager is able to appropriate the entire rent, she maximizes the expected surplus generated by the fund's investments. The surplus also takes into account the manager's income outside the fund. The expected surplus varies with the manager's closing decisions at date 1, represented by $\omega^{H}(\mathcal{A})$, $\omega^{L^\circ}(\mathcal{A})$ and $\omega^{L^-}(\mathcal{A})$. By assumption (A1), η denotes the fund's expected return if the success probability is p, the expected surplus, $S(\omega^H, \omega^{L^\circ}, \omega^{L^-})$, can be written as

$$\begin{aligned} S(\omega^H, \omega^{L^\circ}, \omega^{L^-}) &= -1 - \mathbb{E}(\omega) + \\ &+p\Big(F(\omega^H)(1+R^H)(\eta+1) + (1-F(\omega^H))(1+R^H+\mathbb{E}(\omega|\omega \geq \omega^H))\Big) + \\ &+(1-p)\theta\Big(F(\omega^{L^\circ})(1+R^L)(\eta+1) + (1-F(\omega^{L^\circ}))(1+R^L+\mathbb{E}(\omega|\omega \geq \omega^{L^\circ}))\Big) + \\ &+(1-p)(1-\theta)\Big(F(\omega^{L^-})(1+R^L)(\eta+1-\varepsilon(R^H-R^L)) + (1-F(\omega^{L^-}))(1+R^L+\mathbb{E}(\omega|\omega \geq \omega^{L^-}))\Big). \end{aligned} \tag{2}$$

Optimal Contracting without High Water Mark

First we consider the case in which the manager selects a period performance fee such that the performance-based fee in each period amounts to a constant additional fraction $a \geq 0$ of the gain in fund value during the period and nothing in case of a decrease in fund value. If the first-period fund return is positive, the manager receives a performance fee of $f^H = a(V^H - V_0) = aR^H$ at the end of the first period. If the second-period return is positive, the fee depends on the fund's first period return. In case of a positive first-period return, the manager's second-period fee is $f^{HH} = a(V^{HH} - V^H) = a(1 + R^H)R^H$ upon a positive second-period return. This implies that for the level on manager's expected outside income equal to $apR^H(1 + R^H) := \omega^H(a)$ the manager is indifferent between managing the fund in the second period and launching his expected outside income. In case of a negative first-period return, the second-period fee is $f^{LH} = a(V^{LH} - V^L) = a(1 + R^L)R^H$.[12] Thus, in state L° the manager closes the fund for ω larger than $apR^H(1 + R^L) := \omega^{L^\circ}(a)$ and in state L^- for ω above $a(p - \varepsilon)R^H(1 + R^L) =: \omega^{L^-}(a)$.

Optimal Contracting with High Water Mark

Consider now a fee structure that specifies a linear performance fee, $\widetilde{a} \geq 0$, with a high water mark provision. A high water mark specifies that a performance fee is based on the difference between the fund's value at the end of the period and the historic maximum of fund values provided that this difference is positive. Because the fund value at date 0, $V_0 = 1$ is (trivially) the historic maximum of fund values, the fee level $f^H = \widetilde{a}R^H$ is the same as under a period performance fee. The same applies to V^H and therefore $f^{HH} = \widetilde{a}(1 + R^H)R^H$. If the first-period return is negative, the historic maximum of fund values remains its initial value $V_0 = 1$. This implies that with a high water mark

[12]Note that there is a convexity in the fee structure despite the seemingly linear contract.

f^{LH} is structurally different from its period performance fee counterpart. It is given by $f^{LH} = \max\{0, \widetilde{a}(V^{LH} - 1)\}$, which we assume to be strictly positive. Thus, $f^{LH} = \widetilde{a}((1 + R^L)(1 + R^H) - 1) = \widetilde{a}(R^H(1 + R^L) + R^L) > 0$. For a given value of $\widetilde{a} > 0$, f^{LH} in case of a performance fee with high water mark is strictly smaller than that in the absence of a high water mark.

The corresponding closing thresholds for ω are defined as follows:

$$\omega^H(\widetilde{a}) := \widetilde{a}pR^H(1 + R^H),$$

$$\omega^{L^\circ}(\widetilde{a}) := \widetilde{a}p(R^L + R^H(1 + R^L)),$$

$$\omega^{L^-}(\widetilde{a}) := \widetilde{a}(p - \varepsilon)(R^L + R^H(1 + R^L)).$$

Due to its smaller base, the fee percentage with high water mark $\widetilde{a}$ can be larger than its period performance counterpart a without inducing the manager to continue the fund in states L° and L^-. For a given percentage fee, the manager's optimal closing policy in state H is identical in both performance fee regimes, as the structure of the relevant fee, f^{HH} is not affected by a high water mark.

Comparing the manager's incentive constraints and the resulting expected total surplus levels of the manager between the contracts with and without a high water mark yields a central result:

Proposition 1 *The optimal contract with a high water mark provision yields at least as high a payoff to the fund manager as the optimal contract with a period performance fee.*

Proof: See Appendix A.1.

A contract with a high water mark (weakly) dominates a contract with a period performance fee. The formal argument for this is as follows: By selecting an appropriate fee level, a high water mark contract is able to generate an identical closing policy in the downward states L° and L^- as any given contract with period performance. The fee percentage of the high water mark contract is higher than that of its period performance counterpart. This typically reduces the manager's incentive to close upon a positive first-period return. Only if the optimal period performance contract implies the continuation of the fund with probability one in state H, is it possible that the two types of contracts yield the same payoff to the manager.

Uniform Distribution of Outside Income

To make the benefits of a performance fee with high water mark more transparent, we now assume that the fund manager's outside income is uniformly distributed, i.e. that the manager's outside income ω has cumulative distribution function $F(\omega) = \frac{\omega}{\omega_{max}}$ with density $f(\omega) = \frac{1}{\omega_{max}}$ on $[0, \omega_{max}]$. To allow for a relatively wide spectrum of outside income levels and to simplify the analysis, we also assume that $\omega_{max} = \left(1 + R^H\right)\eta$ (compare (A3)).

The following proposition presents the optimal fee choice dependent on the investor's participation constraint (1) and the manager's closing decision:

Proposition 2 *When the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1 + R^H\right)\eta\right]$*, the optimal contract with a high water mark provision is given by* $\left(\widetilde{a}^*, \widetilde{k}^*\right)$ *with*

$$\widetilde{a}^* = \frac{p^2 R^H (1+R^H)^2 \eta + (1-p)(1+R^L)(R^L + R^H(1+R^L))\left(p\theta\eta + (p-\varepsilon)(1-\theta)(\eta - \varepsilon(R^H - R^L))\right)}{p^3 (R^H(1+R^H))^2 + (1-p)(R^L + R^H(1+R^L))^2\left(p^2\theta + (p-\varepsilon)^2(1-\theta)\right)}$$

and

$$\widetilde{k}^* = \eta - \widetilde{a}^* p R^H.$$

The optimal contract with a high water mark provision specifies a higher performance fee parameter and a lower management fee than the optimal contract with a period performance fee, (a^*, k^*)*.*

From an ex ante perspective, the optimal contract with a high water mark leads to a higher probability of closing in state L^-*, to a lower probability of closing in state* H*, and to a strictly larger payoff to the hedge fund manager than the optimal contract with a period performance fee.*

Proof: See Appendix A.2.

Compared to the optimal contract with a period performance fee, the optimal contract with a high water mark reduces excessive continuation by generating a lower expected fee from continuing the fund when closing is efficient. The fund manager is compensated for this reduction in expected fees by larger fee payment state H. Because in that state closing is not efficient, the contract with high water mark further improves efficiency buy curbing the manager's incentive to close the fund too often. The manager indeed strictly prefers a contract with high water mark to one without whenever it affects her closing decision. A high water mark arrangement is more efficiently able to utilize the superior information of the manager about fund prospects. Thus, the model identifies a rationale for including high water mark provisions in hedge fund management contracts based on closing considerations.

For a numerical example, Figure 3 displays the expected surplus of a hedge fund under both performance fee structures as a function of the performance fee parameter. It illustrates that for any fixed level of the performance fee parameter, the expected surplus with high water mark is larger than with period performance fee. The figure also shows that the optimal performance fee parameter is higher with high water mark than with period performance fee. Adjusting the fixed management fee allows the manager to design the

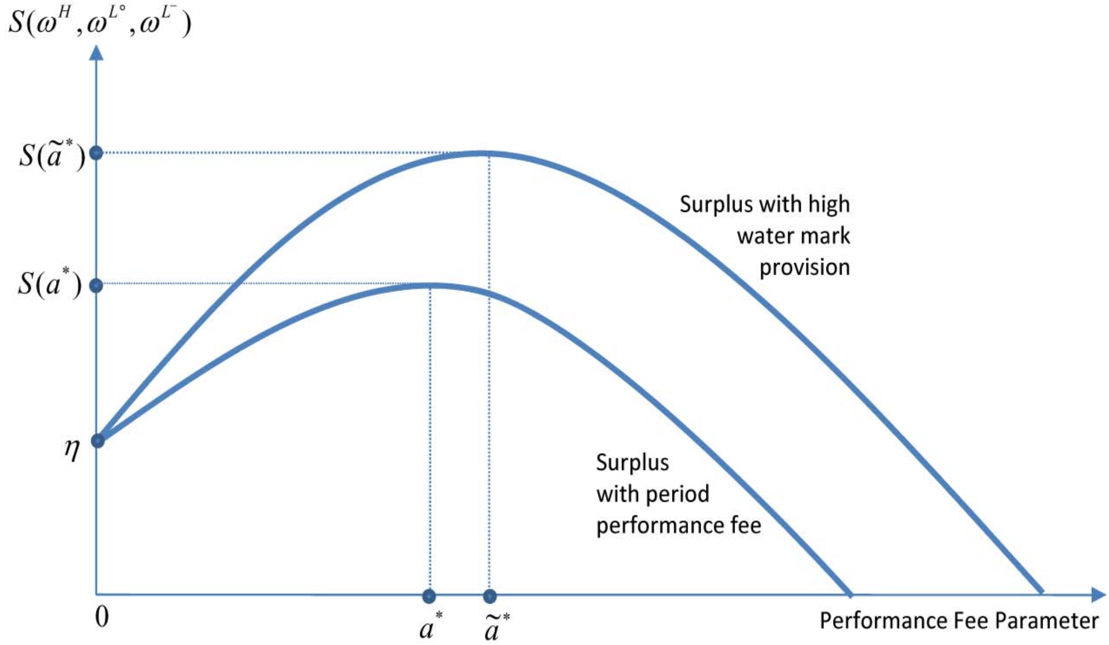

Figure 3: Expected fund surplus in different performance fee regimes.
The surplus-function with high water mark provision has an absolute maximum value $\tilde{a}^*$ and surplus-value $S(\tilde{a}^*)$, that are larger than the corresponding absolute maximum value a^* and surplus-value $S(a^*)$ of the surplus-function with period performance fee.

performance fee structure in a way to optimize incentives. Because the structure of the performance fee with high water mark is better suited to align incentives between the investor and the fund manager, compensation via a management fee is lower than when a period performance fee is used.[13]

[13]Theoretical papers on the use of high water marks tend to ignore management fees. One notable exception is Lan, Wang and Yang (2011) who find that the management fee discourages risk taking by the fund manager.

The numerical examples in Table 1 show that the optimal contract with period performance fee includes a significantly lower performance fee parameter than the optimal contract with high water mark. The latter is better able to control both the incentive to excessively close in the upward state and the incentive to excessively continue upon a deterioration of the fund's prospects. The contract with high water mark generates, however, excessive closing in the downward state when fund prospects remain intact. This holds both relative to the efficient closing policy and relative to the policy generated by the contract with period performance fee. As in this state in the middle of the spectrum the difference in surplus from continuation compared to closing are relatively small, the misaligned incentives between investor and manager are less consequential than the benefits of a high water mark structure in the extreme states.

	Parameter: R^H=7%, R^L=-6.5%, p=0.6, ε=0.3, Θ=0.3 *Pr.*(State *H*)=0.6, *Pr.*(State $L°$)=0.12, *Pr.*(State L^-)=0.28		Parameter: R^H=5%, R^L=-4%, p=0.6, ε=0.2, Θ=0.2 *Pr.*(State *H*)=0.6, *Pr.*(State $L°$)=0.08, *Pr.* (State L^-)=0.32	
	Period Performance Fee	Performance Fee with High Water Mark	Period Performance Fee	Performance Fee with High Water Mark
	a^*=26.99% k^*=0.47% $S(a^*)$=1.92%	$\tilde{a}^*$=38.06% k^*=0.001% $S(\tilde{a}^*)$=2.11%	a^*=36.58% k^*=0.3% $S(a^*)$=1.75%	$\tilde{a}^*$=46.47% k^*=0.006% $S(\tilde{a}^*)$=1.84%
	Closing Probabilities in States (Efficient Probabilities in parentheses):			
H	***29.14% (0%)***	***0.084% (0%)***	***21.62% (0%)***	***0.41% (0%)***
L°	***38.08% (12.62%)***	***99.39% (12.62%)***	***28.33% (8.57%)***	***84.83% (8.57%)***
L^-	***69.04% (100%)***	***99.79% (100%)***	***52.22% (100%)***	***89.93% (100%)***

Table 1: Numerical Examples.
Two different parameter combinations and the corresponding levels of performance fee, management fee, fund's expected surplus and closing probabilities in states *H*, *L°* and L^- in the contracts with period performance fee and performance fee with a high water mark provision, respectively.

4 Implications

The model provides a number of implications on closing behavior, performance and contract design.

a) *Funds with high water marks tend to close more frequently upon negative performance.*

Corollary 1 *When the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1 + R^H\right)\eta\right]$*, the the closing probability upon a negative first-period return is higher for the optimal contract with a high water mark provision than for the optimal contract with a period performance fee.*

Proof: See Appendix A.3.1.

The use of high water mark provisions improves hedge funds' closing decisions. After periods of poor performance, high water marks reduce excessive fund continuation.[14] Empirical studies confirm the impact of high water marks on closure rates of funds. Liang and Park (2010) find that hedge funds with high water marks tend to close more quickly upon bad performance.[15]

b) *Funds with high water marks tend to outperform funds without high water marks after periods of poor performance.*

Corollary 2 *Suppose the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1 + R^H\right)\eta\right]$*. Then, conditional on fund continuation after a negative first-period return, the expected second-period after-fee return of a fund with a high water mark provision exceeds that of a fund with a period performance fee.*

Proof: See Appendix A.3.2.

Contracts with high water marks provide improved incentives for closing by specifying lower expected fees when the fund is under water. Thus, hedge funds with high water marks tend to have better after-fee performance when returns have (recently) been poor relative to otherwise comparable funds with period performance fees.

[14]This implication contrasts with the prediction in Aragon and Qian (2010) that high water marks reduce the probability of fund closing upon negative returns, because higher expected after-fee fund returns reduce investors' incentives to withdraw capital.

[15]The authors, however, don't explicitly test for the statistical difference between the two parameter estimates.

c) *In funds with high water mark contracts a higher expected performance tends to be associated with higher management fees.*

Corollary 3 *Suppose the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1+R^H\right)\eta\right]$ *and the contract contains a high water mark provision. Then, an increase in the level of the positive return,* R^H*, leads to an increase in the management fee,* $\widetilde{k}^*$*.*

Proof: See Appendix A.3.3.

The level of the management fee is fund managers' instrument to extract the surplus the fund generates. As the magnitude of a positive return, R^H, increases, the management fee increases as well.[16] Deuskar et al. (2011) find that successful funds tend to increase their management fees suggesting that these increases of the management fees reflect higher return expectations.

5 Extension

Non-negativity of the management fee and probability of performance deterioration

In the model described above, the performance fee is set to align closing incentives of the fund manager with those of the investor whereas the adjustment of the management fee allows the fund manager to extract the expected surplus. This is possible, because we focus on the parameter space for which the optimal contract specifies a positive management fee. If, however, the requirement that the management fee be non-negative becomes binding, the specified performance fee affects the expected level of cash returns to the investor. Concretely, only a suboptimally low performance fee from an incentive point of view satisfies the investor's participation constraint. In the following, we examine this case maintaining the assumption of the uniform distribution of outside income on $\left[0, \left(1+R^H\right)\eta\right]$ and show that it is present when the likelihood of the deterioration of the fund's prospects is relatively small.

[16] A change in parameters not only affects the magnitude of the fund's expected return but also the optimal performance fee rate and therefore the expected fee income appropriated by the performance fee component alone. Thus, although other surplus-increasing parameter changes, such as increases in R^L or p tend to be associated with higher management fees, there are parameter combinations such that this is not the case.

Proposition 2 and Appendix A.2 reveal that the restriction on the management fee, $k \geq 0$, is binding if and only if the optimal performance fee rate derived in Section 3 exceeds $\frac{\eta}{pR^H}$.[17] It turns out that the optimal performance fee parameter of a contract with period performance fee never exceeds that value (see Appendix A.4). Therefore, we focus in the following on the derivation of the high water mark performance fee parameter, $\widetilde{a}^*$. Due to the convexity of the optimization problem, the new optimal contract has the form $(\widetilde{a}^*, k = 0)$.

The new value of the period performance fee rate with a high water mark provision, $\widetilde{a}^*_{k=0}$, can be derived from the investor's participation constraint (1) and solves the following equation:[18]

$$\begin{aligned} &-(\widetilde{a}^*_{k=0})^2 \frac{(1-p)}{\omega_{max}}(R^L+R^H(1+R^L))^2(p^2\theta+(p-\varepsilon)^2(1-\theta))+ \\ &+\widetilde{a}^*_{k=0}\frac{(1-p)}{\omega_{max}}(1+R^L)(R^L+R^H(1+R^L))\Big(p\theta\eta+(p-\varepsilon)(1-\theta)(\eta-\varepsilon(R^H-R^L))\Big)+ \\ &+(1+p(1+R^H))(\eta-\widetilde{a}^*_{k=0}pR^H) = 0. \end{aligned} \tag{3}$$

The performance fee rate $\widetilde{a}^*_{k=0}$ is smaller than the expected surplus maximizing performance fee rate $\widetilde{a}^*$ derived in Proposition 2.

To gain a better understanding of the possible states in which the performance fee with high water mark provision $\widetilde{a}^*$ increases beyond the threshold $\frac{\eta}{pR^H}$ and its consequences, we relate it to the model parameter θ, which characterizes the probability of a deterioration of the fund's prospects. Recall that in case of a negative first-period return the deterioration of prospects occurs with probability $1-\theta$.

Analyzing the new corresponding optimal contract and expected fund surplus leads to the following result:

Proposition 3 *Suppose the manager's outside income in period* 2 *is uniformly distributed on* $\left[0, \left(1+R^H\right)\eta\right]$. *The surplus maximizing performance fee rate* $\widetilde{a}^*$ *is decreasing in* θ.

There exists a critical value for θ, $\theta^\circ \in (0,1)$, *below which the management fee restriction,* $k \geq 0$, *is binding.*

---

[17] Note that if a performance fee parameter larger than $\frac{\eta}{pR^H}$ is chosen, the probability of fund continuation in state H is equal to one. Thus, a further increase in the performance fee parameter does not affect the closing probability in that state.

[18] The lengthy closed form solution of (3) is presented in Appendix A.4.

There is a critical value of θ, $\widehat{\theta} \in (\theta^{\circ}, 1)$, below which the optimal contract with period performance fee leads to a strictly higher expected surplus than the optimal contract with a high water mark provision.

Proof: See Appendix A4.

	Period Performance Fee	Performance Fee with High Water Mark
θ =0.1	a^* =29.62% k^* =0.27% $S(a^*)$ =0.92%	$\tilde{a}^*$ =42.70% $\tilde{k}^*$ =0.031% $S(\tilde{a}^*)$ =1.03%
θ =0.3	a^* =33.92% k^* =0.19% $S(a^*)$ =0.98%	$\tilde{a}^*$ =43.82% $\tilde{k}^*$ =0.011% $S(\tilde{a}^*)$ =1.04%
θ =0.5	a^* =37.52% k^* =0.12% $S(a^*)$=1.03%	$\tilde{a}^*$ =44.63% $\tilde{k}^*$ =0.00% $S(\tilde{a}^*)$ =1.05%
θ =0.7	a^* =40.60% k^* =0.07% $S(a^*)$ =1.10%	$\tilde{a}^*$ =45.01% $\tilde{k}^*$ =0.00% $S(\tilde{a}^*)$ =1.07%
θ =0.9	a^* =43.25% k^* =0.02% $S(a^*)$=1.16%	$\tilde{a}^*$ =45.51% $\tilde{k}^*$ =0.00% $S(\tilde{a}^*)$ =1.08%

Table 2: Numerical Examples. (Parameters: R^H=3%, R^L=-2.5%, p=0.6, ε=0. 4)
With increasing θ, performance fee parameters $a^*, \tilde{a}^*$ and expected surplus $S(a^*), S(\tilde{a}^*)$ increase; at the same time the management fees $k^*, \tilde{k}^*$ decrease. For $\theta > \hat{\theta}$ the expected surplus in the optimal contract with period performance fee $S(a^*)$ is larger than its counterpart $S(\tilde{a}^*)$.

For a given performance fee rate, a high water mark provision specifies a larger difference in fee income for the manager across states. While this is always beneficial from an incentive standpoint, the additional restriction of a non-negative management fee may significantly impair the manager's desirability of a high water mark provision but not that of a period performance fee. There exist circumstances in which a contract with period performance fee leads to a larger expected surplus and is therefore preferred to a contract with a high water mark. Hence, the choice of a period performance fee may not only be determined by the absence of signaling considerations as shown in Aragon and Qian (2010); it may also be a consequence of the more severe effects of limited liability restrictions on contracts with high water marks.

Contracts with period performance fees are preferred only if the probabilities of the deterioration of funds' prospects are relatively low. It appears reasonable to assume that investors assign significant probabilities of downward adjustments of fund prospects to small funds or those run by managers that lack extensive track records. Thus, the derived result is consistent with findings by Aragon and Qian (2010) that those types of funds more commonly employ high water mark provisions.

6 Robustness of the Results

Intermittent Redemption by the Investor

So far, we have abstracted from allowing the investor to withdraw funds after period 1. Given the linear investment technology, the investor either wants to redeem all or none of his funds. Thus, allowing for the intermittent redemption of funds, the investor has the opportunity to effectively close the fund. In the following, we discuss some of the main aspects of including the investor's option to redeem funds intermittently in the fund management contract. We do this maintaining the assumption of a uniform distribution of outside income as given in Section 3.

First, note that it is never superior to allow the investor to redeem capital intermittently when the fee structure is designed in a way that the option is never exercised. Doing so only introduces additional restrictions.

If there are circumstances in which the investor closes the fund, he does so only upon a negative first-period return and under inferior information than the manager. His information is inferior in two ways: the investor cannot distinguish between states L° and L^{-}, and also is not informed about the realization of ω.

The investor leaves his capital in the fund if doing so increases his expected cash flows. In case of negative first-period return, fund withdrawal yields the investor a cash flow of the date-1 value of the fund, which is given by the fund's gross value, V^{L}.

Consider the case that the investor withdraws his capital from the fund with certainty after a negative first-period return. Note that in this case only the fees f^{H} and f^{HH} are paid with positive probability. Because these fee payments are independent of whether the contract specifies a period performance fee or contains a high water mark provision, no discrimination between these two performance fee structures is necessary. The investor's break even constraint in this case is given by:

$$
\begin{aligned}
&V_0 \leq -k + \\
&+p\Big(-f^H + F(\omega^H)(p(V^{HH} - f^{HH}) + (1-p)V^{HL}) + (1 - F(\omega^H))V^H\Big) + \\
&+(1-p)V^L.
\end{aligned}
$$

Based on this constraint, the maximal level of performance fee parameter a that the investor is willing to accept is obtained if k is set to zero and is equal to $\frac{\eta}{pR^H}$.[19]

The manager anticipates intermittent redemption and consequently the fund's closing by the investor after a negative first-period return. Then, fund's expected surplus is:

$$
\begin{aligned}
S^{rd}(\omega^H) &= -1 - \mathbb{E}(\omega) + \\
&+p\Big(F(\omega^H)(1+R^H)(\eta+1) + (1-F(\omega^H))(1+R^H+\mathbb{E}(\omega|\omega \geq \omega^H))\Big) + \\
&+(1-p)V^L \\
\Leftrightarrow S^{rd}(\omega^H) &= \eta - \mathbb{E}(\omega) + p\Big(F(\omega^H)(1+R^H)\eta + (1-F(\omega^H))\mathbb{E}(\omega|\omega \geq \omega^H)\Big).
\end{aligned}
$$

The manager maximizes the expected fund surplus with respect to her optimal closing policy in state H, which is a function of the performance parameter a. Given assumption (A3), the expected surplus in state H is at least as high as the maximal level of her outside opportunity ω_{max}. Thus, the manager chooses the maximum possible continuation probability $F(\omega^H)$ equal to 1. Then, the optimal performance fee parameter is equal to $a^* = \frac{\eta}{pR^H}$.

Now we are in a position to compare the expected surplus with intermittent redemption by the investor to the one generated by the contract derived in Proposition 2.

Proposition 4 *When the manager's outside income in period 2 is uniformly distributed on $\left[0, \left(1 + R^H\right)\eta\right]$, the optimal contract with intermittent redemption by the investor leads to a strictly lower expected surplus than the optimal contract without intermittent redemption.*

Proof: See Appendix A5.

Given that the manager has private information, granting intermittent redemption rights to the investor is not optimal. Thus, the model implies that intermittent redemption rights are typically used for reasons other than increasing the efficiency of the fund's

[19] For the proof see Appendix A.5.

closing policy. They may, for example, be included because of liquidity needs by hedge fund investors.

Capital Contribution by the Manager

So far, it has been assumed that the manager does not invest own financial wealth in the fund. Actually, hedge fund managers typically do invest their own capital in the fund.[20] The following arguments introduce the case in which the manager possesses financial wealth of $A > 0$ and contributes it to the fund. The initial investment amount that can be invested profitably is $V_0 = A + Y \equiv 1$, where A is the part indicates the manager's contribution and $Y = 1 - A$ investor's, respectively.

If the invested amount yields a positive return $R^H > 0$, the fund's value increases after the first period to V^H. We can distinguish between the manager's $A(1 + R^H)$ and the investor's $(1 - A)(1 + R^H)$ shares, respectively. Analogous is the wealth development after a negative first-period return $R^L < 0$ with decreasing value V^L and the manager's $A(1 + R^L)$ and the investor's $(1 - A)(1 + R^L)$ shares, respectively. This allocation of the share proportions between both parties is also kept constant in the second period. As described in the basic model the investor still has the same participation constraint (1). The modification is that he is now interested in changes in the portion of $(1 - A)V_0$.

The manager's decision whether to operate the fund after the first period or to close it depends on the realization of her outside income ω. In the new model setting the cutoff levels of ω depend not only on the period performance fee arrangement but also on the fund's fraction that was generated by the manager's investment A. Taking into account the adjusted values of $\omega^H, \omega^\circ, \omega^{L^-}$, the manager's closing decision and the expected surplus $S(\omega^H, \omega^\circ, \omega^{L^-})$ remain unaffected, as described (2) in the basic model.

Recall that the optimal contract in case $A = 0$, that was described in proposition 2, is given by $(\widetilde{a}^*, \widetilde{k}^*)$. After comparing the expected total fund surplus levels, with respect to the manager's incentive constraint between the contracts with the period performance fee and the period performance fee with a high water mark provision, we can state the following result:

Proposition 5 *When the manager contributes financial wealth $A > 0$ to the fund invest-*

[20] Agarwal et al. (2009), report that "Discussions with industry practitioners suggest that often the manager reinvests all of the incentive fees earned back into the fund." Thus, they calculate the manager's coinvestment as the cumulative value of the incentive fee reinvested together with the returns earned on it.

ment and her outside income in period 2 is uniformly distributed on $\left[0, \left(1+R^H\right)\eta\right]$ she chooses the optimal contract with the high water mark provision $(\widetilde{a}^*, \widetilde{k}_A^*)$. The optimal performance fee rate $\widetilde{a}^*$ does not depend on A. The optimal management fee $\widetilde{k}_A^*$ and the expected total fund surplus $S(\omega^H, \omega^{L^\circ}, \omega^{L^-})$ increase with increasing A.

From an ex ante perspective, the increase in financial wealth $A > 0$ leads to a lower probability of closing in state H, to a lower closing probability in state L° and to higher closing probability in state L^-.

Proof: See Appendix A6.

Thus the manager investing her own capital in the fund brings about further alignment of interests. It is obvious to see that the higher the financial contribution the higher the expected loss in the case of low return realization. In order to prevent expected loss the manager's incentive to close the fund as efficiently as possible in each of the states increases with the amount of her investment in the fund. Findings by Agarwal et al. (2009), show that higher levels of managerial ownership, in the funds which use incentive contracts with inclusion of high-water mark provision, are associated with superior performance. The numerical examples in Table 3 show the changes of closing probabilities in different states as the managers capital contribution increases.

Because the optimal performance fee rate $\widetilde{a}^*$ is independent of whether the manager contributes financial wealth or not, the main results of Proposition 2 do not change.

7 Conclusion

The paper studies the choice between two different types of performance fee structures in hedge fund management contracts: fees based solely on the performance during the preceding period and fees based on the performance relative to the historical fund value maximum. It provides a rationale for the inclusion of the latter, so-called high water mark provisions, based on the argument that such structures facilitate efficient fund closing. Significant levels of expected fees in states that potentially warrant fund closing provide incentives for fund managers to continue the fund even when doing so is inefficient. Management contracts with high water mark provisions specify lower expected fees after periods of negative performance when fund closing may be warranted. In equilibrium, managers receive higher fee rates and thus higher compensation in case of a continuously positive value development of the fund.

	Closing Probabilities in States (Efficient Probabilities in parentheses):		
	H	***L°***	***L⁻***
A=0	***0.414% (0%)***	***84.825% (8.57%)***	***89.93% (100%)***
A=1%	***0.410%***	***84.062%***	***90.246%***
A=10%	***0.373%***	***77.200%***	***93.507%***
A=20%	***0.331%***	***69.574%***	***97.131%***

Table 3: Numerical Examples.(Parameters: R^H=5%, R^L=-4%, p=0.6, ε=0.2, Θ=0.2). The table schows change in the fund's continuation probabilities with increasing level on managerial capital contri-bution *A* if the manager's optimal contract includes management fee and period performance fee with a high water mark provision .

Our approach implies that funds with high water marks tend to close more quickly upon periods of poor performance than their period performance fee counterparts. If, however, such funds with high water mark arrangements decide to continue, their performance levels on an after-fee basis are expected to be superior to comparable funds employing period performance fees. The model is also consistent with empirical evidence that high water marks are more common in smaller funds and funds run by managers without extensive track records.

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Appendix

We use the following notation for all proofs:

The indicator function of a subset A of a some set X is a function $\mathbb{I}_A : X \to \{0,1\}$ defined as $\mathbb{I}_A(x) := \begin{cases} 1 & if \quad x{\in}A \\ 0 & if \quad x{\notin}A. \end{cases}$

$\phi_{\mathbb{I}} := p^2R^H(1+R^H)^2\eta{+}(1{-}p)(1+R^L)(R^L\mathbb{I}{+}R^H(1{+}R^L))\Big(p\theta\eta{+}(p{-}\varepsilon)(1{-}\theta)(\eta{-}\varepsilon(R^H-R^L))\Big)$

$\varphi_{\mathbb{I}} := p^3(R^H(1{+}R^H))^2{+}(1{-}p)(R^L\mathbb{I}{+}R^H(1{+}R^L))^2\Big(p^2\theta{+}(p{-}\varepsilon)^2(1{-}\theta)\Big)$

$\xi := p(1+R^H)^2 + (1-p)(1+R^L)^2$

For parameters $R^H > 0, R^L \in (\frac{-pR^H}{1-p}, \frac{-(p-\varepsilon)R^H}{1-p+\varepsilon})$, [21] $p \in (0,1)$, $\varepsilon \in (0,p)$ and $\theta \in (0,1)$ it is straightforward that $\phi_{\mathbb{I}}, \varphi_{\mathbb{I}}, \xi > 0$.

A.1 Proof of Proposition 1. Consider a contract with the period performance fee rate $a > 0$ and a contract with a high water mark provision and performance fee rate $\widetilde{a} > 0$, so that the following condition is satisfied:

$$\widetilde{a} = a \underbrace{\frac{R^H(1+R^L)}{(R^L + R^H(1+R^L))}}_{>1, \quad since R^L<0}.$$

Note, that the performance fee rate with a high water mark provision $\widetilde{a}$ is strictly larger than the corresponding period performance fee rate a in this setting. The manger's closing threshholds in state L^- are equal in both regimes: $\omega^{L^-}(a) \equiv \omega^{L^-}(\widetilde{a})$, and also in state L°:

$$\omega^{L^\circ}(\widetilde{a}) = a\frac{R^H(1+R^L)}{(R^L + R^H(1+R^L))}p(R^L + R^H(1+R^L)) = apR^H(1+R^L) = \omega^{L^\circ}(a).$$

Calculating the corresponding closing thresholds in state H yields:

$$\omega^H(\widetilde{a}) = \underbrace{apR^H(1+R^H)}_{=\omega^H(a)} \underbrace{\frac{R^H(1+R^L)}{(R^L + R^H(1+R^L))}}_{>1} > \omega^H(a)$$

Using (1) and comparing the fund's expected surplus under the contract with the period performance fee a and the contract with the performance fee with a high water mark provision $\widetilde{a}$ yields:

$$S(\omega^H, \omega^{L^\circ}, \omega^{L^-})(\widetilde{a}) \geq S(\omega^H, \omega^{L^\circ}, \omega^{L^-})(a)$$

[21] Follows from the assumptions (A1) and (A2).

$$\Leftrightarrow (1+R^H)\eta\Big(F(\omega^H(\widetilde{a}))-F(\omega^H(a))\Big)+(1-F(\omega^H(\widetilde{a})))\mathbb{E}(\omega|\omega\geq\omega^H(\widetilde{a}))-(1-F(\omega^H(a)))\mathbb{E}(\omega|\omega\geq\omega^H(a))\geq 0$$

$$\Leftrightarrow (1+R^H)\eta\underbrace{\Pr(\omega^H(a)\leq\omega<\omega^H(\widetilde{a}))}_{>0,\quad since\quad \omega^H(a)<\omega^H(\widetilde{a})}+\int_{\omega^H(\widetilde{a})}^{\omega_{max}}\omega f(\omega)d\omega-\int_{\omega^H(a)}^{\omega_{max}}\omega f(\omega)d\omega\geq 0$$

$$\Leftrightarrow \int_{\omega^H(a)}^{\omega^H(\widetilde{a})}\Big((1+R^H)\eta-\omega\Big)f(\omega)d\omega>0$$

The last integral is positive for all $\omega\in[0,\,\omega_{max}]$ with $\omega_{max}=(1+R^H)\eta$. ■

A.2 Proof of Proposition 2. Rewrite the equality (2)

$$S(\omega^H,\omega^{L^\circ},\omega^{L^-})=\eta+p\frac{\omega^H}{\omega_{max}}(1+R^H)\eta+(1-p)\theta\frac{\omega^{L^\circ}}{\omega_{max}}(1+R^L)\eta+(1-p)(1-\theta)\frac{\omega^{L^-}}{\omega_{max}}(1+R^L)(\eta-\varepsilon(R^H-R^L))$$

Using definitions for ω^H, ω^{L° and ω^{L^-} we receive: $S(\omega^H(a),\omega^{L^\circ}(a),\omega^{L^-}(a))=\eta+\frac{a_{\mathbb{I}}\phi_{\mathbb{I}}}{\omega_{max}}+\frac{a_{\mathbb{I}}^2}{2\omega_{max}}$. The expected surplus-function, as a parabola, is twice continuously differentiable at $a_{\mathbb{I}}$. The first derivative of the function equals to zero at the extrema:

$$a_{\mathbb{I}}=\frac{\phi_{\mathbb{I}}}{\varphi_{\mathbb{I}}}:=\begin{cases}\widetilde{a}^* & if\quad \mathbb{I}\equiv 1\\ a^* & if\quad \mathbb{I}\equiv 0\end{cases}.$$

The second order condition yields

$$\frac{d^2S(\omega^H,\omega^{L^\circ},\omega^{L^-})}{da_{\mathbb{I}}}=\frac{-1}{\omega_{max}}\Big(p^3(R^H(1+R^H))^2+(1-p)(R^L\mathbb{I}+R^H(1+R^L))^2(p^2\theta+(p-\varepsilon)^2(1-\theta))\Big)$$

and is constant and negative for all parameters in the domain of the definition. This shows that the expected surplus has a unique global maximum at $a_{\mathbb{I}}$.

Because the investor must break even, a maximal possible amount that can be collected in $k_{\mathbb{I}}$ with respect to the investor's participation constraint (1) is equal to: $k_{\mathbb{I}}=\eta-a_{\mathbb{I}}pR^H$. Thus, the optimal contract with a high water mark provision is given by $\left(\widetilde{a}^*,\widetilde{k}^*\right)$ and optimal contract with period performance fee is given by (a^*,k^*) .

Lemma 1 *The performance fee rate a^* in the optimal contract with period performance fee is smaller than the performance fee rate $\widetilde{a}^*$ in the optimal contract with high water mark provision $\widetilde{a}^*$. The management fee k^* is larger than $\widetilde{k}^*$.*

Proof: We show that $\widetilde{a}^*=\frac{\phi_1}{\varphi_1}>\frac{\phi_0}{\varphi_0}=a^*$. Recall that $\varphi_{\mathbb{I}}>0$, so we have

$0 \quad < \quad \phi_1\varphi_0-\phi_0\varphi_1$

$$\Leftrightarrow \underbrace{(R^L+2R^H(1+R^L))}_{>R^H(1+R^L)}\Big(\theta+(1-\frac{\varepsilon}{p})^2(1-\theta)\Big) > R^H(1+R^L)\Big(\theta+(1-\frac{\varepsilon}{p})(1-\theta)(1-\frac{\varepsilon}{\eta}(R^H-R^L))\Big)$$

$$\Leftrightarrow \theta+(1-\frac{\varepsilon}{p})^2(1-\theta) >\theta+(1-\frac{\varepsilon}{p})(1-\theta)(1-\frac{\varepsilon}{\eta}(R^H-R^L))$$

$$\Leftrightarrow (1-\frac{\varepsilon}{p}) > (1-\frac{\varepsilon}{\eta}(R^H-R^L))$$

$$\Leftrightarrow 0 \gneqq R^L \to True$$

With $\widetilde{a}^* > a^*$ we can argue: $\widetilde{a}^* pR^H > a^* pR^H \Leftrightarrow \eta - \widetilde{a}^* pR^H < \eta - a^* pR^H \Leftrightarrow \widetilde{k}^* < k^*$. For $k_{\mathbb{I}} \equiv 0$, we have $0 = \eta - a_{\mathbb{I}} pR^H \Leftrightarrow a_{\mathbb{I}} = \frac{\eta}{pR^H}$, thus the restriction on management fee, $k \geq 0$, is binding iff the optimal performance fee rate exceeds the value $\frac{\eta}{pR^H}$. ■

In the next step we show that from the ex ante perspective, the optimal contract with a high water mark provision leads to strictly lower closing probability in state H. Pr(state H) is equal to1 $- F(\omega^H(a))$, hence it is sufficient to show that: $F(\omega^H(a^*)) = \frac{\omega^H(a^*)}{\omega_{max}} < \frac{\omega^H(\widetilde{a}^*)}{\omega_{max}} = F(\omega^H(\widetilde{a}^*)) \Leftrightarrow \omega^H(a^*) < \omega^H(\widetilde{a}^*)$:

$$\omega^H(\widetilde{a}^*) \equiv \omega^H(a^*\omega^H(\widetilde{a}^*) \equiv \widetilde{a}^* pR^H(1+R^H) > a^* pR^H(1+R^H) \equiv \omega^H(a^*).$$

We show now that from ex ante perspective, the optimal contract with a high water mark provision leads to higher closing probabilities in states L° and L^-. Proof for state L°:

$$\begin{aligned} \omega^{L^\circ}(a^*) &> \omega^{L^\circ}(\widetilde{a}^*) \\ \Leftrightarrow a^* pR^H(1+R^L) &> \widetilde{a}^* p(R^L + R^H(1+R^L)) \\ \Leftrightarrow \phi_0\varphi_1 R^H(1+R^L) &> \phi_1\varphi_0(R^L + R^H(1+R^L)) \\ \Leftrightarrow \underbrace{R^H(1+R^L)}_{>0}\underbrace{(\phi_0\varphi_1 - \phi_1\varphi_0)}_{>0} &> \underbrace{\phi_0\varphi_1}_{>0}\underbrace{R^L}_{<0} \end{aligned}$$

Analogously for state L^-:

$$\begin{aligned} \omega^{L^-}(a^*) &> \omega^{L^-}(\widetilde{a}^*) \\ a^*(p-\varepsilon)R^H(1+R^L) &> \widetilde{a}^*(p-\varepsilon)(R^L + R^H(1+R^L)) \end{aligned}$$

use the same proof as in state L°. ■

A.3 Proofs of Corollaries 1, 2 and 3.

A.3.1 **Proof of Corollary 1**: With proof of Proposition 2 we have: $\omega^{L^\circ}(a^*) > \omega^{L^\circ}(\widetilde{a}^*)$. Thus, the closing probability upon a negative first-period return in state L° for the optimal contract with high water mark provision is larger than the corresponding probability for the optimal contract with period performance fee: $1 - F(\omega^{L^\circ}(\widetilde{a}^*)) > 1 - F(\omega^{L^\circ}(a^*))$.

Analogously we have $1-F(\omega^{L^-}(\widetilde{a}^*)) > 1-F(\omega^{L^-}(a^*))$ in state L^-. Thus, the weighted closing probability upon a negative first-period return for the optimal contract with high water mark provision is higher than the corresponing weighted closing probability for the optimal contract with period performance fee:

$$\theta(1-F(\omega^{L^\circ}(\widetilde{a}^*)))+(1-\theta)(1-F(\omega^{L^-}(\widetilde{a}^*))) > \theta(1-F(\omega^{L^\circ}(a^*)))+(1-\theta)(1-F(\omega^{L^-}(a^*)))$$

$$\Leftrightarrow \theta\Big(\underbrace{F(\omega^{L^\circ}(a^*))-F(\omega^{L^\circ}(\widetilde{a}^*))}_{>0}\Big)+(1-\theta)\Big(\underbrace{F(\omega^{L^-}(a^*)-F(\omega^{L^-}(\widetilde{a}^*))}_{>0}\Big) > 0 \qquad \blacksquare$$

A.3.2 **Proof of Corollary 2**: In the first step we use Proposition 2 and Corollary 1 to see: $f^{LH}(a^*) = a^*R^H(1+R^L) > \widetilde{a}^*(R^L+R^H(1+R^L)) = f^{LH}(\widetilde{a}^*)$. Conditional on fund continuation after a negative first-period return, the investor has a posterior belief to face state L° equal to $\theta F(\omega^{L^\circ})/(\theta F(\omega^{L^\circ})+(1-\theta)F(\omega^{L^-}))$, that is independent of whether the contract specifies a high water mark provision or a period performance fee, because

$$\begin{aligned}
\frac{\theta\omega^{L^\circ}(\widetilde{a}^*)}{\theta\omega^{L^\circ}(\widetilde{a}^*)+(1-\theta)\omega^{L^-}(\widetilde{a}^*)} &= \frac{\theta\omega^{L^\circ}(a^*)}{\theta\omega^{L^\circ}(a^*)+(1-\theta)\omega^{L^-}(a^*)}\\
\Leftrightarrow \omega^{L^\circ}(\widetilde{a}^*)\omega^{L-}(a^*) &= \omega^{L^\circ}(a^*)\omega^{L^-}(\widetilde{a}^*)\\
\Leftrightarrow \frac{p}{2p-\varepsilon} &= \frac{p}{2p-\varepsilon}.
\end{aligned}$$

In the second step we compare the expected fund's second-period after-fee return in both regimes:

$$\begin{aligned}
\frac{\theta F(\omega^{L^\circ}(\widetilde{a}^*))p(V^{LH}-\widetilde{a}^*(R^L+R^H(1+R^L)))}{\theta F(\omega^{L^\circ}(\widetilde{a}^*))+(1-\theta)F(\omega^{L^-}(\widetilde{a}^*))} &> \frac{\theta F(\omega^{L^\circ}(a^*))p(V^{LH}-a^*R^H(1+R^L))}{\theta F(\omega^{L^\circ}(a^*))+(1-\theta)F(\omega^{L^-}(a^*))}\\
\Leftrightarrow V^{LH}-\widetilde{a}^*(R^L+R^H(1+R^L)) &> V^{LH}-a^*R^H(1+R^L)\\
\Leftrightarrow a^*R^H(1+R^L) &> \widetilde{a}^*(R^L+R^H(1+R^L)). \qquad \blacksquare
\end{aligned}$$

A.3.3 **Proof of Corollary 3**: The manager's outside income in period 2 is uniformly distributed on $\left[0,\left(1+R^H\right)\eta\right]$. Thus the manager chooses the optimal contract with the high water mark provision given by $\left(\widetilde{a}^* = \frac{\phi_1}{\varphi_1}, \widetilde{k}^* = \eta - \widetilde{a}^*pR^H\right)$. The management fee can be rewritten as $\widetilde{k}^* = pR^H+(1-p)R^L-\widetilde{a}^*pR^H = pR^H(1-\widetilde{a}^*)+(1-p)R^L$. Recall that $\widetilde{a}^*$ depends on R^H, so the first derivative of $\widetilde{k}^*$ equals:

$$\frac{d\widetilde{k}^*(R^H)}{dR^H} = \frac{d}{dR^H}pR^H(1-\widetilde{a}^*)+\frac{d}{dR^H}(1-p)R^L = p\Big(\underbrace{(1-\widetilde{a}^*)+\frac{R^H}{\varphi_1}(\widetilde{a}^*\frac{d\varphi_1}{dR^H}-\frac{d\phi_1}{dR^H})}_{>0}\Big) > 0$$

because $\widetilde{a}^* = \frac{\phi_1}{\varphi_1} < 1 < \frac{1-R^H\frac{d\phi_1}{dR^H}}{1-R^H\frac{d\varphi_1}{dR^H}}$ and $\frac{d\varphi_1}{dR^H} < \frac{d\phi_1}{dR^H}$. For a numerical example see Figure 4.

$\blacksquare$

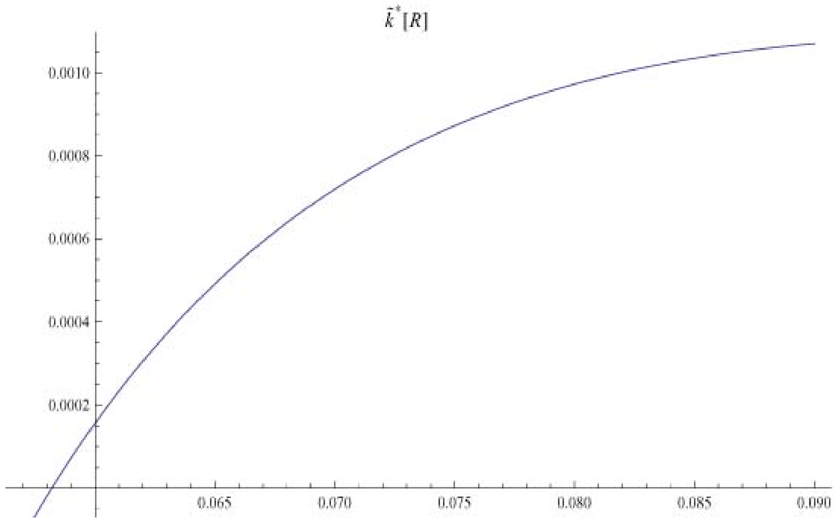

Figure 4: Numerical Example. (Parameters: ***R^L=-5.5%, p=5.5, ε=0. 3, Θ=0.2***)
Changes in management fee for varying values of ***R^H***.